Examlet 4a **Foundations of Advanced Math** **4/12/24**

1. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $x \sim y \Leftrightarrow x - y \equiv_3 3$. Determine whether $\sim$ is an equivalence relation.

2. Let $S = \{a, b, c, d, e\}$, and let $\sim = \{(a, a), (b, b), (b, d), (b, e), (c, c), (d, b), (d, d), (d, e), (e, b), (e, d), (e, e)\}$
.

(a) Give the equivalence classes of $\sim$.

(b) Give the partition associated with $\sim$.

3. Let S be a set and Π a partition of S. Let $\sim$ be a relation on S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$.

   (a) Show $\sim$ is a reflexive relation.

   (b) Show $\sim$ is a symmetric relation.

   (c) Show $\sim$ is a transitive relation.

4. Regarding the function $f : A \to B$ as a subset of $A \times B$,

   (a) State the definition of f being injective.

   (b) State the definition of f being surjective.

5. Call two vertices v_1 and v_2 in a graph G **barely connected** iff there exists a walk from v_1 to v_2, but there exists an edge in G such that if that edge were removed, then there no longer exists a walk from v_1 to v_2. Determine whether the relation of being barely connected is reflexive, symmetric, and transitive.