Problem Set 10 **Foundations** **Due 4/28/24**

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. How many strings of up to 5 numbers are there that use (actually use) only the numbers 1, 4, and 5?
2. If a jar contains five balls, one red, two blue, and two white, and two balls are drawn at random from the jar (without replacement), what is the probability that the red ball is drawn if you know that not both balls drawn were blue?
3. $\forall x, y \in N, x + (y + 0) = (x + y) + 0$
4. $\forall x, y, z \in N, x + (y + z) = (x + y) + z \Rightarrow x + (y + z') = (x + y) + z'$
5. $\forall x, y, z \in N, x + (y + z) = (x + y) + z$
6. $\forall y \in N, 0 + y = y + 0$
7. $\forall x, y \in N, x + y = y + x \Rightarrow x' + y = y + x'$
8. $\forall x, y \in N, x + y = y + x$
9. Using the definition of $S(A)$ from section 5.6, write $S(\varnothing), S(S(\varnothing)), S(S(S(\varnothing)))$, and $S(S(S(S(\varnothing))))$ explicitly. How many elements are in each of these sets?
10. With the understanding that $0' = 1, 1' = 2, 2' = 3$, and $3' = 4$, where these are elements of a Peano system, show that $2 + 2 = 4$.