Problem Set 3 **Foundations** **Due 2/12/24**

Four of these problems will be graded (our choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but you must write up your own final submission without reference to any sources other than the textbook and instructor.

The Fibonacci numbers are defined by the two-term recurrence relationship

$$F_1 = 1, \;\; F_2 = 1, \;\; F_{n+2} = F_{n+1} + F_n \;\text{ for }\; n \in \mathbb{Z}^+$$

1. Find F_3 through F_{10}.
2. $F_1 + F_2 + ... + F_n = F_{n+2} - 1$ for every $n \in \mathbb{Z}^+$.

3. Do the Chapter 1 Followup assignment on WeBWorK, available via

   https://webwork.coe.edu/webwork2/MTH-215/

4. Consider the formula $1 + 2 + 3 + ... + n = \frac{n^2+n+1}{2}$.
   (a) Write the formula in sigma notation.
   (b) Show that if this formula works for $n = k$, then it also must work for $n = k + 1$.
   (c) Explain why mathematical induction does not prove that this formula is true for all $n \in \mathbb{N}$.
5. Prove that if A has n elements, then $\mathcal{P}(A)$ has 2^n elements. [Hint: Induction!]
6. For any sets A and B, $(A \cap B)' = A' \cup B'$.
7. If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$.
8. Show that $A - (B \cap A) = (A - B) \cap A$.
9. Show that
$$\left(\bigcup_{i \in I} A_i \right)' = \bigcap_{i \in I} A_i'$$
10. Show that
$$A \cup \bigcap_{i \in I} B_i = \bigcap_{i \in I} (A \cup B_i)$$