Problem Set 8 **Foundations** **Due 4/8/23**

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but you must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Express the definition of the quotient of two functions in terms of ordered pairs.
2. Express the definition of the composition of two functions in terms of ordered pairs.
3. Express the definition of an injective function in terms of ordered pairs.
4. Any graph where every vertex has odd degree must have an even number of vertices.
5. Any graph where each vertex has degree at least 2 is connected.
6. Any graph where some vertex v_0 is joined to each other vertex in the graph by a walk is connected.
7. The maximum possible number of edges in a graph with n vertices is _____.
8. We say that two vertices v_1 and v_2 of a graph G are **adjacent** $\Leftrightarrow \exists$ an edge $\{v_1, v_2\}$ in G. Determine whether the relation of being adjacent in a graph is reflexive, symmetric, and transitive.
9. We say that two vertices v_1 and v_2 of a graph G are **in the same component of** G $\Leftrightarrow \exists$ a walk from v_1 to v_2 in G. Determine whether the relation of being in the same component of a graph is reflexive, symmetric, and transitive.
10. We say that two vertices v_1 and v_2 of a graph G are **on a common cycle of** $G \Leftrightarrow \exists$ a cycle including v_1 and v_2. Determine whether the relation of being on a common cycle of a graph is reflexive, symmetric, and transitive.