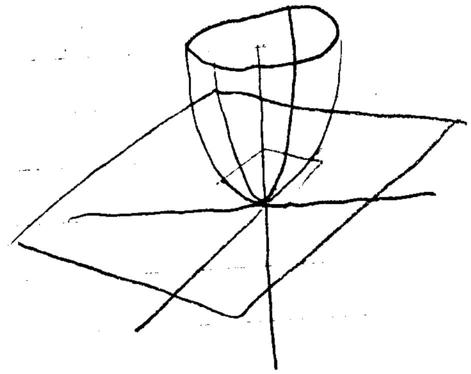
1) $x+y+z=24$

$z=24-x-y$

$z=x^2+y^2$



5/5

$x^2+y^2=24-x-y$

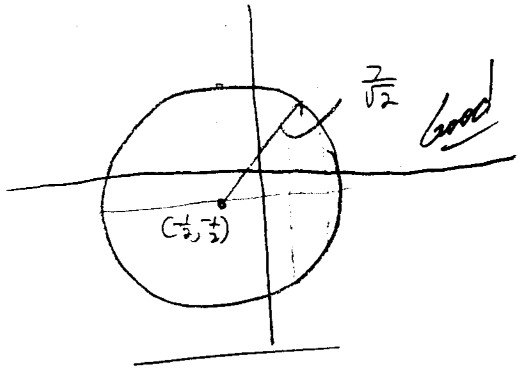
$x^2+x+y^2+y=24$

$(x^2+x+\frac{1}{4})+(y^2+y+\frac{1}{4})=24+\frac{1}{4}+\frac{1}{4}$

$(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=\frac{49}{2}$

$(-\frac{1}{2},-\frac{1}{2})=C$

$\frac{7}{\sqrt{2}}=r$



Good

$(y+\frac{1}{2})^2=\frac{49}{2}-(x+\frac{1}{2})^2$

$\underline{y=\pm\sqrt{\frac{49}{2}-(x+\frac{1}{2})^2}-\frac{1}{2}}$

$x: \frac{-7}{\sqrt{2}}-\frac{1}{2}, \quad \frac{7}{\sqrt{2}}-\frac{1}{2}$

$\frac{-7\sqrt{2}-1}{2}, \quad \frac{7\sqrt{2}-1}{2}$

In[7]:= **Integrate[24 - x - x^2 - y - y^2, {x, (-1 - 7 $\sqrt{2}$) / 2, (7 $\sqrt{2}$ - 1) / 2},
{y, (- $\sqrt{97/4 - x^2 - x}$ - 1 / 2), ($\sqrt{97/4 - x^2 - x}$ - 1 / 2)}]**

$MaxExtraPrecision ::meprec : In increasing internal precision while attempting to evaluate
$-\frac{14+\sqrt{2}}{2\sqrt{2}} + \frac{1}{2}\left(1+7\sqrt{2}\right)$, the limit $MaxExtraPrecision = 49.9999999999999911` was
reached. Increasing the value of $MaxExtraPrecision may help resolve the uncertainty.

$MaxExtraPrecision ::meprec : In increasing internal precision while attempting to evaluate
$\frac{1}{2}\left(1-7\sqrt{2}\right) - \frac{-14+\sqrt{2}}{2\sqrt{2}}$, the limit $MaxExtraPrecision = 49.9999999999999911` was
~~reached.~~ ~~In~~creasing the value of $MaxExtraPrecision may help resolve the uncertainty.

Out[7]= $\frac{2401\,\pi}{8}$

In[5]:= **Integrate[1, {x, (-7 / $\sqrt{2}$ - 1 / 2), (7 / $\sqrt{2}$ - 1 / 2)},
{y, (- $\sqrt{49/2 - (x + 1/2)^2}$ - 1 / 2), ($\sqrt{49/2 - (x + 1/2)^2}$ - 1 / 2)},
{z, (x^2 + y^2), (24 - x - y)}]**

$MaxExtraPrecision ::meprec : In increasing internal precision while attempting to
evaluate $\frac{1}{2} + \frac{7}{\sqrt{2}} - \frac{14+\sqrt{2}}{2\sqrt{2}}$, the limit $MaxExtraPrecision = 49.9999999999999911`
was reached. Increasing the value of $MaxExtraPrecision may help resolve the uncertainty.

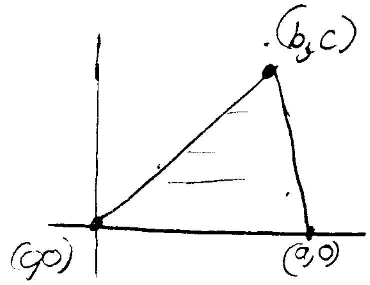
$MaxExtraPrecision ::meprec : In increasing internal precision while attempting to
evaluate $\frac{1}{2} - \frac{7}{\sqrt{2}} - \frac{-14+\sqrt{2}}{2\sqrt{2}}$, the limit $MaxExtraPrecision = 49.9999999999999911`
was reached. Increasing the value of $MaxExtraPrecision may help resolve the uncertainty.

Out[5]= $\frac{2401\,\pi}{8}$

5/5

2) $y = \frac{c}{b} x$
$x = \frac{b}{c} y$

$y = \frac{c}{b-a}(x-a)$
$\frac{y(b-a)}{c} = x - a$
$x = \frac{y(b-a)}{c} + a$



Good

$$m = \int_0^c \int_{\frac{by}{c}}^{\frac{y(b-a)}{c}+a} \rho \, d\cancel{x}\, dy.$$

$$\rho \int_0^c \left[\frac{y(b-a)}{c} + a - \frac{by}{c} \right] dy$$

$$\rho \left[\frac{c^2(b-a)}{2c} + ac - \frac{bc^2}{2c} \right]$$

$$\rho \left[\tfrac{1}{2}(cb - ca) + ac - \frac{bc}{2} \right]$$

$$\rho \left[\cancel{\frac{cb}{2}} - \frac{ca}{2} + ca - \cancel{\frac{cb}{2}} \right]$$

$$m = \rho \frac{ca}{2}$$

$$\bar{y} = \frac{2}{\rho ca} \int_0^c \int_{\frac{by}{c}}^{\frac{y(b-a)}{c}+a} \rho y \, dx\, dy$$

$$\frac{2}{ca} \int_0^c y \left[\left(\frac{y(b-a)}{c} + a \right) - \left(\frac{by}{c} \right) \right] dy$$

$$\frac{2}{ca} \int_0^c \left[\frac{y^2(b-a)}{c} + ya - \frac{by^2}{c} \right] dy$$

$$\frac{2}{ca} \left[\frac{c^3(b-a)}{3\cancel{c}} + \frac{c^2 a}{2} - \frac{bc^3}{3\cancel{c}} \right]$$

$$2 \left[\frac{c(b-a)}{3a} + \frac{c}{2} - \frac{bc}{3a} \right]$$

$$\cancel{\frac{2cb}{3a}} - \frac{2ca}{3a} + c - \cancel{\frac{2bc}{3a}}$$

$$\left[-\frac{2c}{3} + c \right] \Rightarrow \frac{1}{3} c = \bar{y}$$

$$\bar{x} = \frac{2}{\rho ca} \int_0^c \int_{\frac{by}{c}}^{\frac{y(b-a)}{c}+a} \rho x \, dx\, dy.$$

$$\frac{1}{ca} \int_0^c \left[\left(\frac{y(b-a)}{c} + a \right)^2 - \left(\frac{by}{c} \right)^2 \right] dy$$

$$\frac{1}{ca} \int_0^c \left[\frac{y^2(b-a)^2}{c^2} + \frac{2ya(b-a)}{c} + a^2 - \frac{b^2 y^2}{c^2} \right] dy.$$

$$\frac{1}{ca} \left[\frac{c^3(b-a)^2}{3c^2} + \frac{c^2 a(b-a)}{c} + a^2 c - \frac{b^2 c^3}{3c^2} \right]$$

Great

$$\frac{(b-a)^2}{3a} + \frac{a(b-a)}{a} + a - \frac{b^2}{3a}$$

$$\cancel{\frac{b^2}{3a}} - \frac{2ba}{3a} + \frac{a^2}{3a} + b - \cancel{a} + \cancel{a} - \cancel{\frac{b^2}{3a}}$$

$$\left(\frac{a}{3} + \frac{b}{3} \right) \Rightarrow \frac{1}{3}(a+b) = \bar{x}$$

3).a.

$z = \frac{-120}{50} r \sin\theta$ (population function)

$\frac{-120}{50}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\int_0^{50\sin(3\theta)} (r^2 \sin\theta)drd\theta$ = population $= \frac{50625\sqrt{3}}{2} = \boxed{43843}$.

5/5

b.

$-\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\int_0^{50\sin(3\theta)} (r)drd\theta$ = size $= \frac{625\pi}{3} = 654.5$

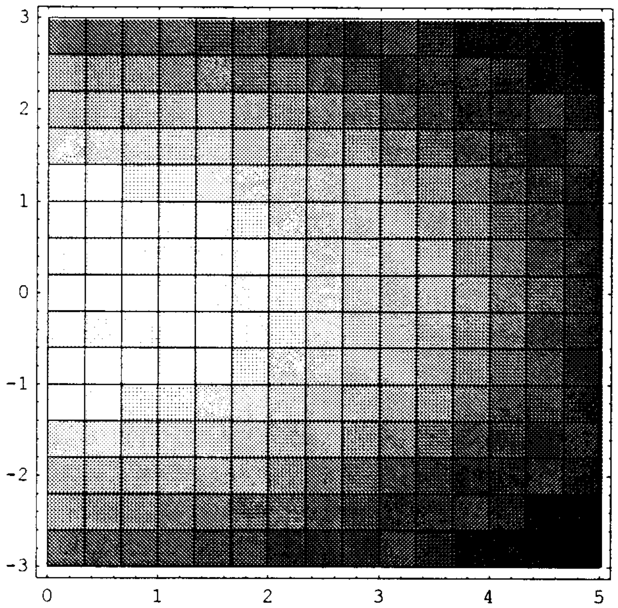
avg $= \frac{\text{pop}}{\text{size}} = \frac{43843}{654.5} =$ $\boxed{\text{66.987 per sq. mile}}$

GAC 18

4. Suppose the temperature at a point (x, y, z)
in a room which measures five meters (from 0 to 5) along the x - axis
by six meters (from - 3 to 3) along the y
axis by 3 meters (from - 3 to 0) along the z axis is given by the function
T (x, y, z) = 68 + 6 e^ (-.02 x^2 - .05 y^2 - .1 z^2) .
a) Produce a graphic representation of the temperature in the room :

In[52]:= **DensityPlot[68 + 6 * E^ (-0.02 * x^2 - 0.05 * y^2), {x, 0, 5}, {y, -3, 3}, PlotRange -> {68, 74}]**

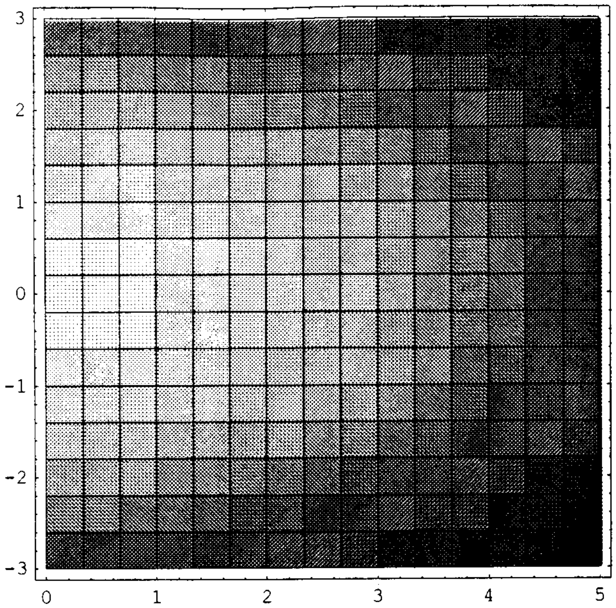
5/5



Out[52]= - DensityGraphics -

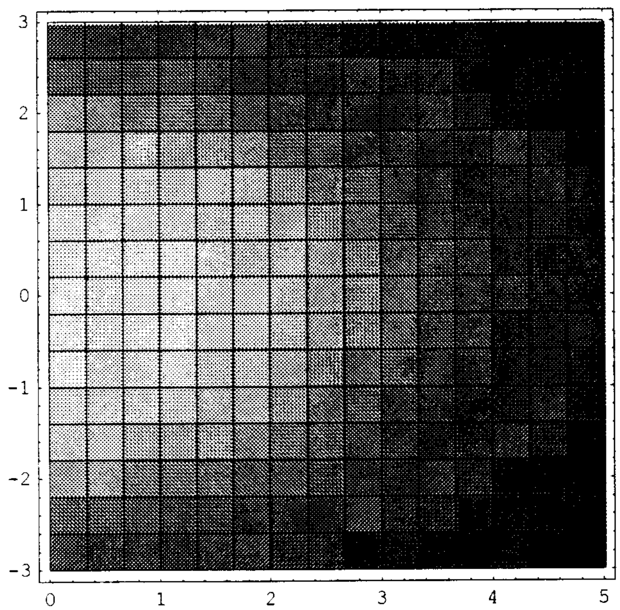
GAC M_2

```
In[47]:= DensityPlot[68 + 6 * E^(-0.02 * x^2 - 0.05 * y^2 - .1), {x, 0, 5}, {y, -3, 3},
          PlotRange -> {68, 74}]
```




```
Out[47]= - DensityGraphics -
```

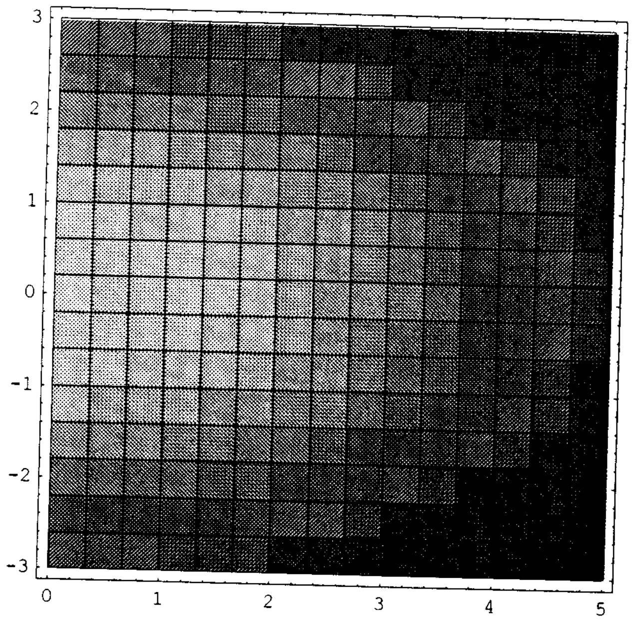
```
In[54]:= DensityPlot[68 + 6 * E^(-0.02 * x^2 - 0.05 * y^2 - .15625), {x, 0, 5}, {y, -3, 3},
          PlotRange -> {68, 74}]
```




```
Out[54]= - DensityGraphics -
```

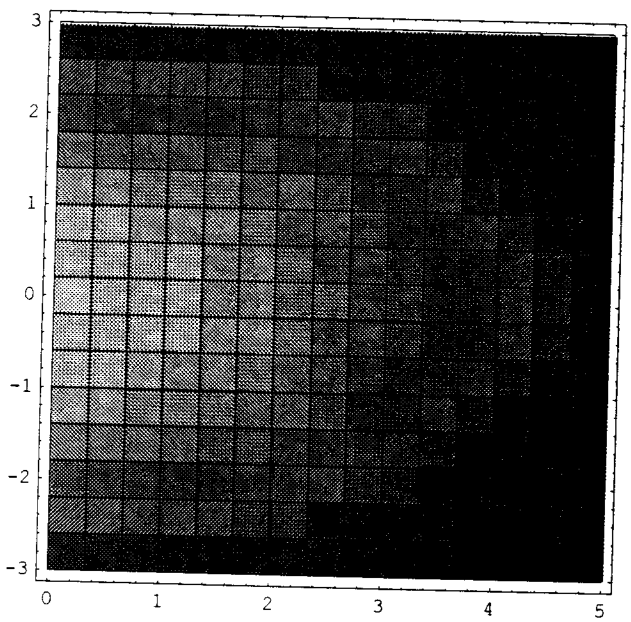
GAC ∂x_3

```
In[56]:= DensityPlot[68 + 6*E^(-0.02*x^2 - 0.05*y^2 - .225), {x, 0, 5}, {y, -3, 3},
  PlotRange -> {68, 74}]
```



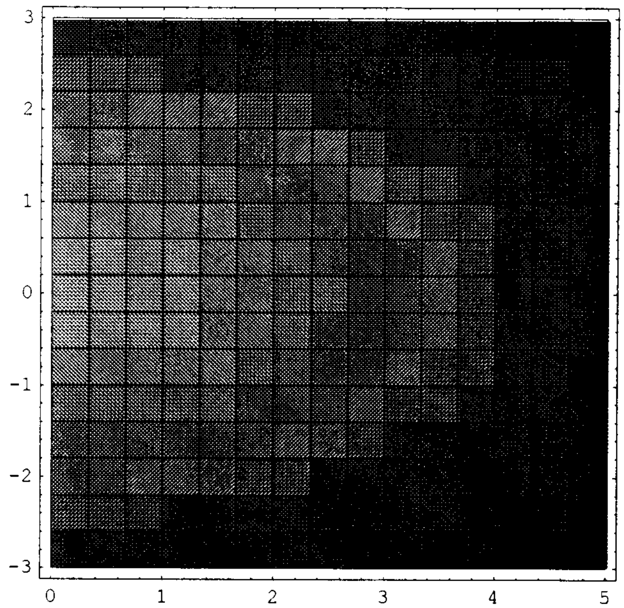
```
Out[56]= - DensityGraphics -
```

```
In[57]:= DensityPlot[68 + 6*E^(-0.02*x^2 - 0.05*y^2 - .30625), {x, 0, 5}, {y, -3, 3},
  PlotRange -> {68, 74}]
```



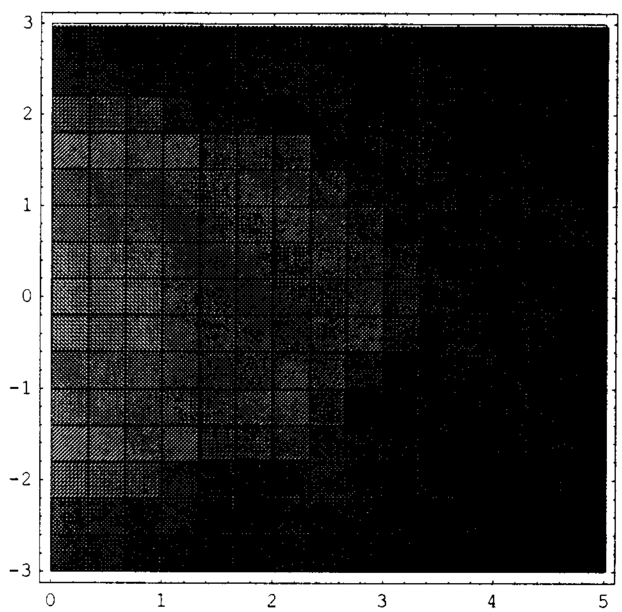
```
Out[57]= - DensityGraphics -
```

```
In[58]:= DensityPlot[68 + 6 * E^(-0.02 * x^2 - 0.05 * y^2 - .4), {x, 0, 5}, {y, -3, 3},
          PlotRange -> {68, 74}]
```



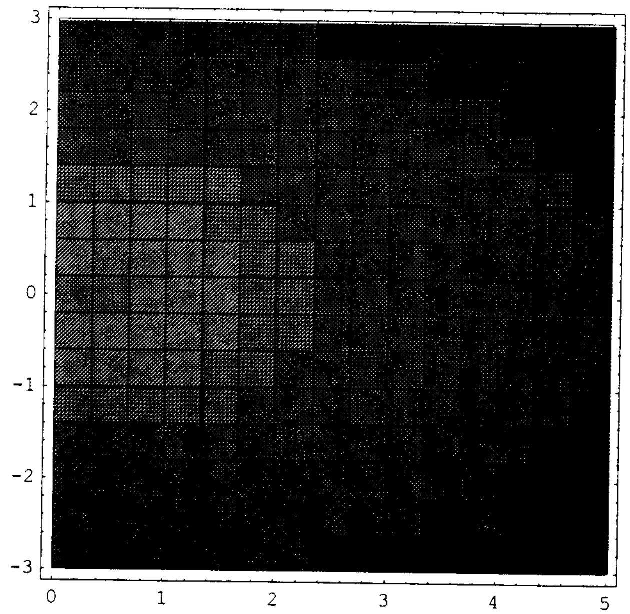
```
Out[58]= - DensityGraphics -
```

```
In[59]:= DensityPlot[68 + 6 * E^(-0.02 * x^2 - 0.05 * y^2 - .50625), {x, 0, 5}, {y, -3, 3},
          PlotRange -> {68, 74}]
```



```
Out[59]= - DensityGraphics -
```

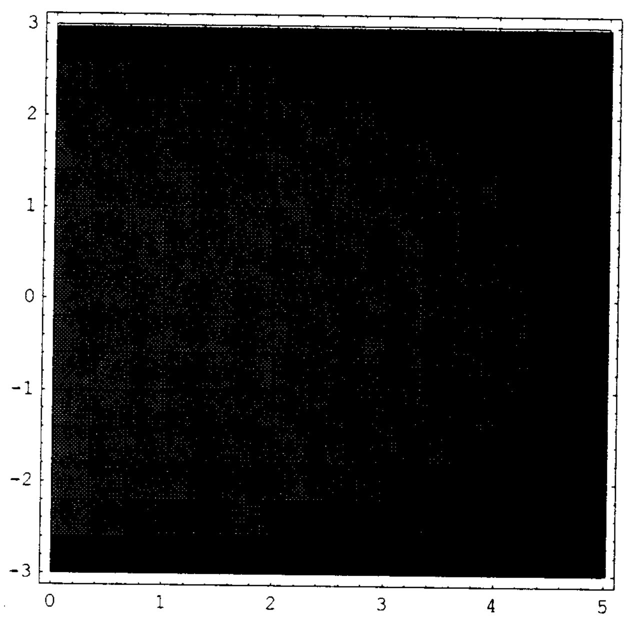
```
In[60]:= DensityPlot[68 + 6*E^(-0.02*x^2 - 0.05*y^2 - .625), {x, 0, 5}, {y, -3, 3},
  PlotRange -> {68, 74}]
```



```
Out[60]= -DensityGraphics-
```

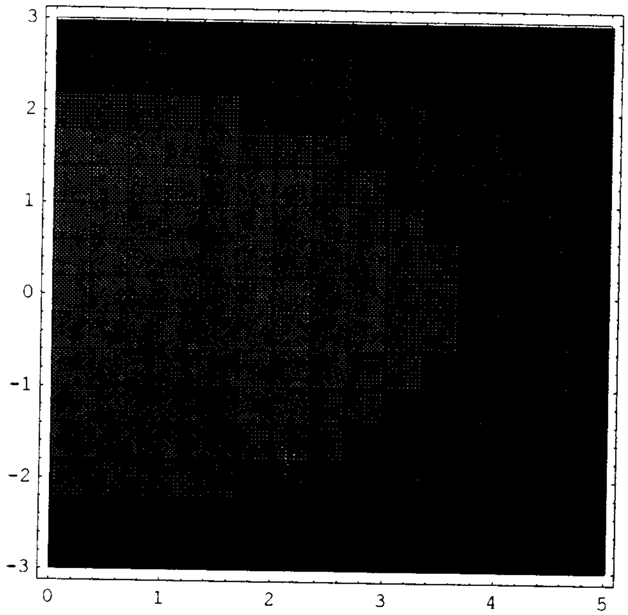
These are great

```
In[61]:= DensityPlot[68 + 6*E^(-0.02*x^2 - 0.05*y^2 - .75625), {x, 0, 5}, {y, -3, 3},
  PlotRange -> {68, 74}]
```



```
Out[61]= -DensityGraphics-
```

In[62]:= DensityPlot[68 + 6 * E^(-0.02 * x^2 - 0.05 * y^2 - .9), {x, 0, 5}, {y, -3, 3},
PlotRange -> {68, 74}]



Out[62]= - DensityGraphics -

b) What sort of physical situation might produce such a temperature distribution?
Such a distribution might be caused by a heat source on the ceiling such as a heater vent blowing into the room or a fire on the floor in the room above that is most intense at x = 0 and is slowly spreading across the floor so the ceiling of the room below is hot and the heat trickles down from there. Or even possibly an air conditioning vent on the floor at x = 5 blowing cold air up into the room.

Great

c) What is the average temperature in the room to the nearest tenth of a degree?

In[64]:= Integrate[Integrate[
Integrate[68 + 6 * E^(-0.02 * x^2 - 0.05 * y^2 - .1 * z^2), {z, -3, 0}], {y, -3, 3}],
{x, 0, 5}]

Out[64]= 6427.41

GAC 24

In[66]:= **6427.41 / (5 * 6 * 3)**

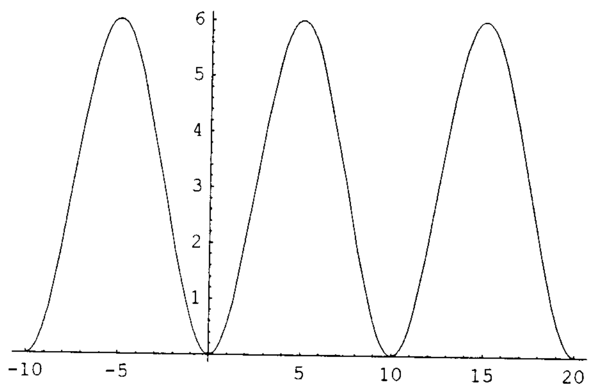
Out[66]= 71.4157

So the average temperature in the room is 6427.41 divided by the volume of the room is 71.4 degrees F.

d) If we adjust the function to T (x, y, z, t) =
68 + 6 * Sin (πt / 10) * E^ (-0.02 * x^2 - 0.05 * y^2 - .1 * z^2)
what is the average temperature in the room to the nearest tenth of a degree?

Since the sin function is cyclic we really only need to figure t to the point where the cycle repeats itself.

In[67]:= **Plot[6 * (Sin[Pi * t / 10]) ^2, {t, -10, 20}]**



Good

Out[67]= - Graphics -

So the range of t is between 0 and 10.

In[69]:= **Integrate[Integrate[Integrate[Integrate[**
68 + 6 * (Sin[Pi * t / 10]) ^2 * E^ (-0.02 * x^2 - 0.05 * y^2 - .1 * z^2), {t, 0, 10}],
{z, -3, 0}], {y, -3, 3}],
{x, 0, 5}]

Out[69]= 62737.

So the average temperature in the room would now be : 62737 / (5 * 6 * 3 * 10),
the integral divide by volume * time

In[72]:= **62737 / (5 * 6 * 3 * 10)**

Out[72]= $\frac{62737}{900}$

Which equals 69.7 degrees F.

Excellent

Vastar

General Purpose Worksheet

Subject	Page No. 6 of 6
File	By / Date

Problem 5: The portion of the surface $z = \frac{h}{a}\sqrt{x^2+y^2}$ $(a, h > 0)$ between the xy-plane & $z = h$ is a rt circular cone of ht h and radius a. Use a surface integral to show that the total surface area of this cone is $S = \pi a\sqrt{a^2+h^2}$

5/5

Surface area $A(s) = \iint_D \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}\; dA$

$$\frac{\partial z}{\partial x} = \frac{h}{a}\left(\frac{1}{2}\right)(2x)(x^2+y^2)^{-1/2} = hx(x^2+y^2)^{-1/2}/a$$

$$\frac{\partial z}{\partial y} = \frac{h}{a}\left(\frac{1}{2}\right)(2y)(x^2+y^2)^{-1/2} = hy(x^2+y^2)^{-1/2}/a$$

$$A = \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{+\sqrt{a^2-x^2}} \sqrt{\left(\frac{hx(x^2+y^2)^{-1/2}}{a}\right)^2 + \left(\frac{hy(x^2+y^2)^{-1/2}}{a}\right)^2 + 1}\; dy\,dx$$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{+\sqrt{a^2-x^2}} \sqrt{\frac{h^2x^2}{a^2(x^2+y^2)} + \frac{h^2y^2}{a^2(x^2+y^2)} + 1}\; dy\,dx$$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{+\sqrt{a^2-x^2}} \sqrt{\frac{h^2}{a^2\cancel{(x^2+y^2)}}\cancel{(x^2+y^2)} + 1}\; dy\,dx$$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{+\sqrt{a^2-x^2}} \sqrt{h^2/a^2 + 1}\; dy\,dx$$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \sqrt{\frac{h^2}{a^2}\left(1+\frac{a^2}{h^2}\right)}\; dy\,dx$$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \frac{1}{a}\sqrt{a^2+h^2}\; dy\,dx$$

$$= \frac{1}{a}\sqrt{a^2+h^2}\int_0^{2\pi}\int_0^a 1\, r\,dr\,d\theta$$

$$= \frac{1}{a}\sqrt{a^2+h^2}\int_0^{2\pi}\frac{1}{2}a^2\,d\theta$$

$$= \frac{1}{a}\sqrt{a^2+h^2}\left(\frac{1}{2}a^2(2\pi)\right)$$

$$\boxed{A_S = a\pi\sqrt{a^2+h_2}}$$

Very Nice

VR10120

+4

Extra Credit: Find surface area of $x^{2/3} + y^{2/3} + z^{2/3} = 1$. Solve for z: $z = (1 - x^{2/3} - y^{2/3})^{3/2}$

Then find z_x and z_y :

$$\frac{\partial z}{\partial x} = \frac{-\sqrt{1 - x^{\frac{2}{3}} - y^{\frac{2}{3}}}}{x^{\frac{1}{3}}} \qquad \frac{\partial z}{\partial y} = \frac{-\sqrt{1 - x^{\frac{2}{3}} - y^{\frac{2}{3}}}}{y^{\frac{1}{3}}}$$

Then, using the surface area formula for one octant:

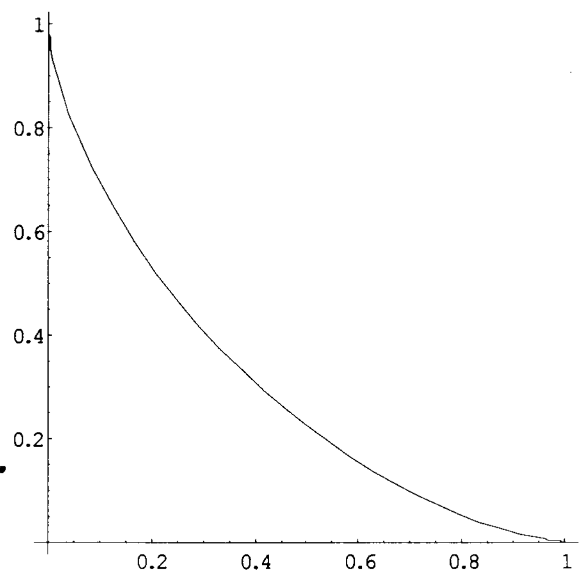
$$S = \iint_A \sqrt{f_x^2 + f_y^2 + 1}\, dA = \int_0^1 \int_0^{\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}} \sqrt{\frac{1 - x^{\frac{2}{3}} - y^{\frac{2}{3}}}{x^{\frac{2}{3}}} + \frac{1 - x^{\frac{2}{3}} - y^{\frac{2}{3}}}{y^{\frac{2}{3}}} + 1}\, dydx$$

Mathematica 3.0 refuses to solve this integral in its current form, so I tried Mathcad 7.0 Professional and got immediate results. Here is the result:

$$\int_0^1 \int_0^{\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}} \sqrt{\frac{1 - x^{\frac{2}{3}} - y^{\frac{2}{3}}}{x^{\frac{2}{3}}} + \frac{1 - x^{\frac{2}{3}} - y^{\frac{2}{3}}}{y^{\frac{2}{3}}} + 1}\, dy\, dx = 0.557$$

This is a reasonable answer when visualizing the amount of area for one octant based on the values of x & y when z = 0:

And so, the moment we've all been waiting for: 8 * .557 = 4.456 is the surface area of the cuboid.



I think I have to reserve the full +5 for the hypothetical exact value (or proof it doesn't exist), but you've done everything short of that. Well done!