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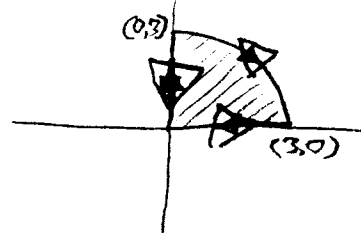
Calculus IV Quiz 5 Spring 1999 4/12/99

1. Compute  $\oint_C y^2 dx + xy dy$  for the path C consisting of the first-quadrant portion of a circle (centered at the origin) of radius 3 traversed ~~counterclockwise~~ clockwise, along with the line segments from (0,0) to (3,0) and from (3,0) to (0,0). *Blame it on Jan!*

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$$\iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$
$$\iint_D (y - 2xy) dA = \iint_D -xy dA$$



$$= \int_0^{\frac{\pi}{2}} \int_0^3 -r^2 \sin \theta dr d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[ -\frac{r^3}{3} \sin \theta \right]_0^3 d\theta$$
$$= -\int_0^{\frac{\pi}{2}} 9 \sin \theta d\theta$$
$$= +9 \cos \theta \Big|_0^{\frac{\pi}{2}}$$
$$= +9(0-1) = -9$$

Answer = -9

2. Compute  $\text{div}(\sin x \mathbf{i} + \cos x \mathbf{j} - z^2 \mathbf{k})$ .

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$$\text{div} = \frac{\partial \sin x}{\partial x} + \frac{\partial \cos x}{\partial y} - \frac{\partial z^2}{\partial z}$$

div = cos x + 0 - 2z



3. Compute  $\text{curl}(x \mathbf{i} + e^y \sin z \mathbf{j} + e^y \cos z \mathbf{k})$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & e^y \sin z & e^y \cos z \end{vmatrix}$$

$$= (e^y \cos z - e^y \cos z) \mathbf{i} + (0-0) \mathbf{j} + (0-0) \mathbf{k}$$

curl =  $\vec{0}$