Calculus IV Exam I Summer 2000 6/16/2000

Each problem is worth 10 points. Show all work for full credit. Please circle all answers and keep your work as legible as possible. The Independent Student Voice of the University of Oklahoma.

1. Find $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+xy+y^2}$ or show that the limit does not exist.

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$x=0$, $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+xy+y^2} = \frac{-y^2}{y^2} = -1$

$y=0$, $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+xy+y^2} = \frac{x^2}{x^2} = 1$

$x=y$, $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+xy+y^2} = \frac{x^2-x^2}{x^2+x^2+x^2} = \frac{0}{3x^2} = 0$

$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+xy+y^2}$ Does Not Exist, since $-1 \neq 1 \neq 0$

Good

W

2. Find the directional derivative of f(x,y) = ln(x-y) at the point (2,1) in the direction of the vector <3,-4>.

FIRST MUST FIND $\vec{u}$ FROM $\vec{V}\langle 3,-4\rangle = \sqrt{3^2+(-4)^2} = \sqrt{25} = 5$

SO $\vec{u} = \frac{1}{5}\langle 3,-4\rangle = \langle \frac{3}{5}, \frac{-4}{5} \rangle$

$\nabla f = \langle f_x, f_y \rangle$

$f_x = \frac{1}{(x-y)} \cdot 1 = \left(\frac{1}{x-y}\right)$

$f_y = \frac{1}{(x-y)} \cdot -1 = \left(-\frac{1}{x-y}\right)$

$\nabla f = \langle \frac{1}{x-y}, \frac{-1}{x-y} \rangle$ OR AT POINT (2,1) $\langle 1,-1 \rangle$

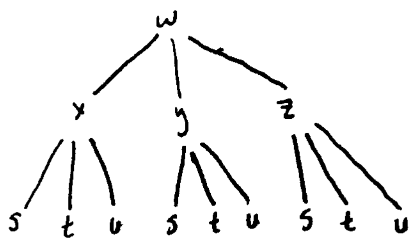
SO

$D = \nabla f \cdot \vec{u} = \langle 1,-1 \rangle \cdot \langle \frac{3}{5}, \frac{-4}{5} \rangle$

$\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$

Great

3. If we have w = f(x,y,z) where x = x(s,t,u), y = y(s,t,u), and z=z(s,t,u), write out the appropriate chain rule for $\frac{\partial w}{\partial s}$.



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

4. Find $\frac{\partial z}{\partial x}$ for the relation $x^{2/3} + y^{2/3} + z^{2/3} = 1$.

W

$\Rightarrow$ Take derivative of 2 sides of the equation with respect to x :

$$\Rightarrow \frac{2}{3} x^{-1/3} + \frac{2}{3} \cdot z^{-1/3} \frac{\partial z}{\partial x} = 0.$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{-\frac{2}{3} x^{-1/3}}{+\frac{2}{3} z^{-1/3}}$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{\frac{1}{\sqrt[3]{x}}}{\frac{1}{\sqrt[3]{z}}} = \boxed{-\sqrt[3]{\frac{z}{x}}}$$

5. Moe the moose has been wandering around studying derivatives of multivariate functions. One day he was walking across the Dartmouth campus when he noticed an area shaped almost exactly like the function $m(x,y) = 0.1x^2+0.2y^2$. If Moe is standing at the point (-2,3), in which direction does the ground rise most steeply and what is that steepness?

An actual news story appearing in *The Oklahoma Daily*:

Moose on campus scares Dartmouth students

HANOVER, N.H. -- Amid the normal flurry of activity on the Dartmouth College Green Thursday morning, students were surprised to find a special visitor running through campus -- a moose.

The moose ran through the West side of the campus Thursday morning.

Although no one was hurt in the incident, there are some safety concerns when moose are running through populated areas, according to Safety and Security Officer Rebel Roberts, who responded to the call.

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DIRECTION = ∇M

Steepness = $|\nabla M| = \sqrt{M_x^2 + M_y^2}$

$M = 0.1x^2 + 0.2y^2$

$M_x = 2(0.1)x$

$M_y = 2(0.2)y$

$M_{x(-2,3)} = 2(0.1)(-2) = -.4$

$M_y(-2,3) = 2(0.2)(3) = 1.2$

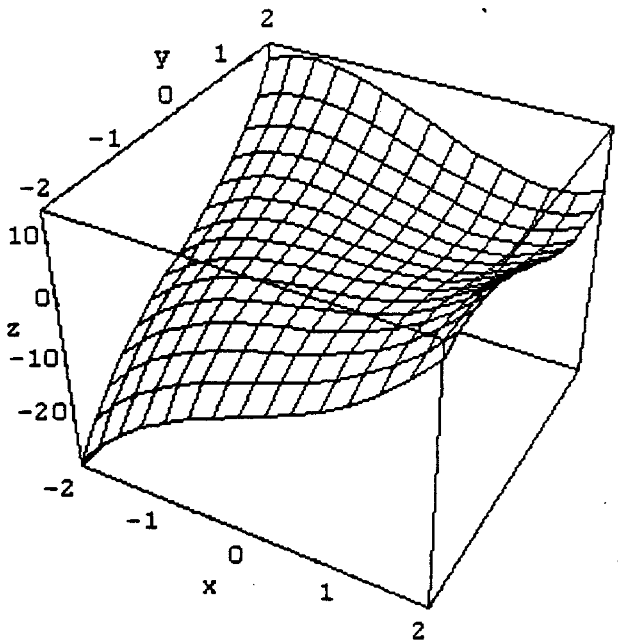
$\boxed{\text{DIRECTION} = \langle -.4, 1.2 \rangle}$

STEEPNES = $|\nabla M| = \sqrt{M_x^2 + M_y^2} = \sqrt{(-.4)^2 + (1.2)^2}$

$\boxed{= \sqrt{1.6}}$

Nice

6. Find all critical points of the function $f(x,y) = x^3 - 3xy + y^3$ and classify them as maxima, minima, or saddle points. The graph provided can serve as a guide, but it's up to *you* to demonstrate things.



I. $f_x(x,y) = 3x^2 - 3y$

$f_y(x,y) = -3x + 3y^2$

II. $3x^2 - 3y = 0$

$3x^2 = 3y$

$x^2 = y$

$-3x + 3y^2 = 0$

$-3x + 3x^4 = 0$

$-3x(1 - x^3) = 0$

$x = 0,\ x = 1$

critical points: $(0,0)$, $(1,1)$

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III. $f_{xx}(x,y) = 6x$

$f_{xy}(x,y) = -3$

$f_{yy}(x,y) = 6y$

$D = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

$D = (6x)(6y) - (-3)^2$

For $(0,0)$: $D = (6)(0)(6)(0) - 9 = -9$

If $D < 0$, then $f(a,b)$ is a saddle point

$\boxed{\text{saddle point: } (0,0)}$

Great

For $(1,1)$: $D = (6x)(6y) - (-3)^2$

$D = (6)(1)(6)(1) - (-3)^2 = 27$

If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local min.

$\boxed{\text{minimum } (1,1)}$

7. Ken is a Calc IV student from California who's getting frustrated with his class. Ken says "Dude, they talk about these, like, *partial* everythings. It's like, they never tell you about the *whole* derivative. But I figure it's gotta be pretty simple, like, if you just add the partial with inspect to x and the partial with inspect to y, that must get you the total, right?"

Help Ken out by explaining (in a way he can understand!) what he's actually getting by his method, and why it's actually better to have two partials than one of his "totals".

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Dude, it's, like, totally simple... Dude. You can't combine 'em like that man. Cuz, with your total, nobody knows whats going on with "inspect" to x and y, Ya know. Like, two partials tell me totally whats going on, while your "total" tells me stuff? Totally Bogus.

Translation...

Ken, my good man, please allow me to help you. What your total gives is merely the change in the overall direction whatever that may mean. It would be favorable to have the partial derivatives since they describe z's behavior with respect to either axis in a clear fashion.

Great

8. Show that the gradient of the function f(x,y) = ln(x-y) is the same at every point on the line y=x-2.

$f_x = 1/x-y \quad f_y = -1/x-y$

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$\nabla f = \langle 1/x-y \, , -1/x-y \rangle$

$\nabla f(x, x-2) = \left\langle \frac{1}{x-(x-2)}, \frac{-1}{x-(x-2)} \right\rangle = \langle 1/2, -1/2 \rangle$

so, since $y = x-2$ x-2 may be substituted in the gradient for y resulting in a gradient composed of constants which is ∴ unvarying along the line $y = x-2$

Nicely Done!

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9. If $f(x,y)=\sqrt[3]{x^3+y^3}$, find $f_x(0,0)$.

$f(x,y) = (x^3+y^3)^{1/3}$

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^3+y^3]^{1/3} - (x^3+y^3)^{1/3}}{h}$$

$$f_x(0,0) = \lim_{h\to 0} \frac{[(0+h)^3+0^3]^{1/3} - (0^3+0^3)^{1/3}}{h}$$

$$= \lim_{h\to 0} \frac{(h^3)^{1/3}}{h}$$

$$= 1$$

Excellent

10. Show that the plane tangent to the surface $f(x,y)=x^2-y^2$ at the point (x_0,y_0,z_0) has $-z_0$ as its z-intercept.

$f_x = 2x$

$f_y = -2y$

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$z - z_0 = 2x_0(x - x_0) - 2y_0(y - y_0)$

$z - z_0 = 2x_0x - 2x_0^2 - 2yy_0 + 2y_0^2$

$z - z_0 = -2x_0^2 + 2y_0^2$

$z - z_0 = -2(x_0^2 - y_0^2)$

$z - z_0 = -2(z_0)$

$z = -2z_0 + z_0$

$\boxed{z = -z_0}$

Wonderful!