1. Evaluate the integral $\int x \cos x \, dx$.

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx = \boxed{x \sin x + \cos x + C}$$

Let $u = x$, $dv = \cos x \, dx$

Then $du = dx$, $v = \sin x$

Check: $\frac{d}{dx}\left(x \sin x + \cos x + C \right) = x \cos x + \sin x - \sin x = x \cos x$ ✓

2. Evaluate the integral $\int x \cos(x^2) \, dx$.

$$\int x \cos(x^2) \, dx$$

LET $u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$$\int \cos(u) \cdot \frac{1}{2} \, du$$

$$\frac{1}{2} \int \cos(u) \, du$$

$$\frac{1}{2} \sin(u) + C$$

$$\boxed{\frac{1}{2} \sin(x^2) + C}$$