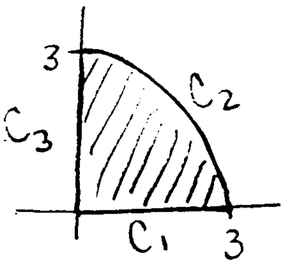
Calculus IV Quiz 7 Summer 2000 7/20/2000

Green's Theorem (provided the proper conditions apply):

$$\oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

1. Use Green's Theorem to compute $\oint_C y\,dx - x\,dy$ where C is the first-quadrant arc of a circle (centered at the origin) of radius 3 oriented counterclockwise followed by the line segment from (0,3) to (0,0) and then the line segment from (0,0) to (3,0).



$$\int_0^{\frac{\pi}{2}} \int_0^3 (-1 - 1) r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \int_0^3 -2r\,dr\,d\theta =$$

$$-\int_0^{\frac{\pi}{2}} r^2 \Big|_0^3 d\theta = -\int_0^{\frac{\pi}{2}} 9\,d\theta = -9\theta \Big|_0^{\frac{\pi}{2}} = -\frac{9\pi}{2} \quad C_2$$

2. Compute the curl of the vector field **F**(x,y,z) = xy **j** + xyz **k**.

$$\text{curl} = \nabla \times \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \langle 0, xy, xyz \rangle$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & xy & xyz \end{vmatrix} = (xz - 0)\hat{i} - (yz - 0)\hat{j} + (y - 0)\hat{k}$$

$$= \boxed{\langle xz, -yz, y \rangle}$$

3. Compute the divergence of the vector field **F**(x,y,z) = x^3**i** - e^{xy}**j** + sin y **k**.

$$\nabla \cdot \vec{F} = \left\langle \frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz} \right\rangle \cdot \langle x^3, -e^{xy}, \sin y \rangle$$

$$= 3x^2 + -xe^{xy} + 0$$

$$= 3x^2 - xe^{xy}$$