1.> Compute: $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

* Solution 1: L'H rule:

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{3x^2}{1} = 3$$

Very nice!

* Solution 2: $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)} = \lim_{x \to 1} (x^2+x+1)$

$\boxed{2}$ If $f(x) = kx^3 - \sin \pi x + e^{\alpha x}$, find $f'(x)$

Solution: $f(x) = kx^3 - \sin \pi x + e^{\alpha x}$

$$\boxed{f'(x) = 3kx^2 - \pi \cos \pi x + \alpha e^{\alpha x}}$$

3. If $y = x\tan x - \frac{\ln x}{x^2} + \cos^3 x$ find $\frac{dy}{dx}$

multiplication rule

quotient rule $\rightarrow (\ln x)(x^{-2})$ change to multiplication

chain rule

$(1)\tan x + \sec^2 x(x) - \frac{1}{x}(x^{-2}) + -2x^{-3}(\ln x) +$

$-3\cos^2 x(\sin x)$

2

$$\frac{dy}{dx} = \tan x + x\sec^2 x - \frac{1}{x^3} + \frac{2}{x^3}\ln x - 3\cos^2 x(\sin x)$$

Good.

2 ④ $T(x,y) = 100/(1 + x^2 + 2y^2)$

For the region where the temperature is at least 20 degrees:

$100/(1+x^2+2y^2) \geq 20 \rightarrow 1 + x^2 + 2y^2 \leq 5 \rightarrow x^2 + 2y^2 \leq 4 \rightarrow \frac{x^2}{4} + \frac{y^2}{2} \leq 1$

The said region is an ellipse whose center is at the 'origin', and whose x- and y- intercepts are $(\pm 2, 0)$ and $(0, \pm\sqrt{2})$, respectively.

Excellent

5, Consider a cylinder with radius r, and suppose water falls into the cylinder at a rate of 1 liter per minute, Give a formula for $d(r,t)$ the depth of the water after t minutes.

Cylinder $= \pi r^2 d$ ← depth $\quad V = \pi r^2 d \quad \frac{1}{\pi r^2} = d \quad d = \frac{1}{\pi r^2}$

rate = 1 liter/minute $\quad d(r,1)$

when $r = 1m \quad d = \frac{1}{\pi(1)^2} \quad d = \frac{1}{\pi}$ when $r = 2m \quad d = \frac{1}{4\pi}$

when $r = 3m \quad \frac{1}{\pi(3)^2} \quad d = \frac{1}{9\pi}$ when $r = 4m \quad \frac{1}{\pi(4)^2} = \frac{1}{16\pi}$

when $r = 0.5m \quad \frac{1}{\pi(0.5)^2} \quad d = \frac{1}{0.25\pi}$ when $r = 0.2m \quad \frac{1}{\pi(0.2)^2} = \frac{1}{0.04\pi}$

at t = 1 minute

radius	depth
0.2m	$1/0.04\pi$
0.5m	$1/0.25\pi$
1 m	$1/\pi$
2 m	$1/4\pi$
3 m	$1/9\pi$
4 m	$1/16\pi$

at r = 1m

time	depth
0.2 min	$0.2/\pi$
0.5 min	$0.5/\pi$
1 min	$1/\pi$
2 min	$2/\pi$
3 min	$3/\pi$
4 min	$4/\pi$

2

radius increases depth decreases

time increases depth increases

$$d = \frac{1t}{\pi r^2}$$

d = depth
t = time
r = radius

time is directly proportional to depth
radius is inversely proportional to depth

Excellent.