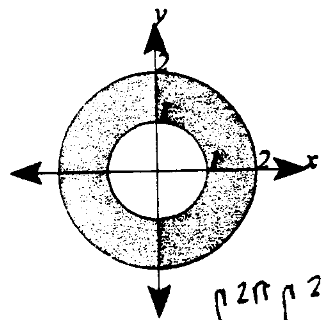
**Quiz 3 Calc IV (Math 2443-003) 6/29/2001**

Each problem is worth 5 points. For full credit indicate clearly how you reached your answer.

1. Set up $\iint_R f(x,y)\,\cancel{dA}$ for each of the regions shown:

5

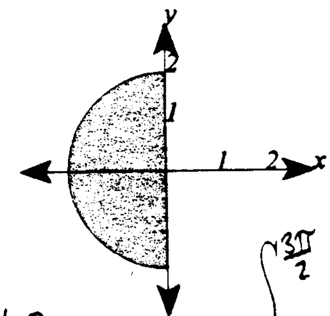
a)




$$\int_0^{2\pi}\int_1^2 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Nice!

b)




$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^2 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

2. ***Set up*** a double integral in polar coordinates for the volume of the solid above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$. $\Rightarrow z^2 = 1-(x^2+y^2)$ (1)

$z=\sqrt{x^2+y^2}$ (2) and (1) is above (2) $\Rightarrow$ (1) $\Rightarrow z=\sqrt{1-(x^2+y^2)}$ $(z>0)$

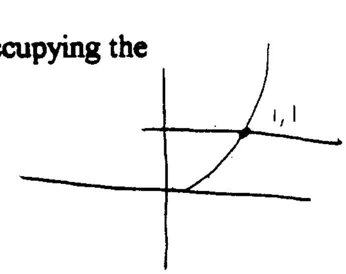
Top view: $\sqrt{x^2+y^2} = \sqrt{1-(x^2+y^2)} \Rightarrow x^2+y^2 = \frac{1}{2} = r^2$

$\Rightarrow r = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\Rightarrow V = 4\int_0^{\frac{\pi}{2}}\int_0^{\frac{\sqrt{2}}{2}} \left(\sqrt{1-r^2} - r\right) r\,dr\,d\theta$$

Nice!

3. Set up integrals for the center of ~~mass of the lamina~~ with density $\rho(x,y) = xy$ occupying the region in the first quadrant bounded by the parabola $y = x^2$ and the line $y = 1$.




$$m = \int_0^1\int_{x^2}^1 xy\,dy\,dx \qquad \bar{x} = \frac{M_y}{m}$$

$$M_y = \int_0^1\int_{x^2}^1 x^2y\,dy\,dx \qquad \bar{y} = \frac{M_x}{m}$$

$$M_x = \int_0^1\int_{x^2}^1 xy^2\,dy\,dx$$

Excellent