Quiz 5 Calc IV (Math 2443-003) 7/16/2001

1. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y)=(2x-y)\mathbf{i}+(3y^2-x)\mathbf{j}$ where C is the top half of the spiral with polar equation $r = \theta/\pi$ beginning at (0,0) and ending at (-1,0) [the coordinates of these points are given in standard rectangular coordinates].

$f(x,y)= x^2-xy+y^3 \qquad f_x=2x-y \qquad f_y=-x+3y^2$

$$\int_C \vec{F}\cdot d\vec{r} = (x^2-xy+y^3)\Big|_{(0,0)}^{(-1,0)} = (1+0+0)-(0-0+0)=\boxed{1}$$

2. Compute $\int_C \langle 2x+y, 3y^2\rangle\cdot d\mathbf{r}$ for C the line segment beginning at (1,0) and ending at (2,3).

$x(t)= 1+t$

$y(t)= 3t$

$\vec{r}(t)=\langle 1+t, 3t\rangle \qquad 0\le t\le 1$

$F(\vec{r}(t)) = \langle 2(1+t)+3t,\ 3(3t)^2\rangle = \langle 2+5t, 27t^2\rangle$

$\vec{r}\,'(t)=\langle 1,3\rangle$

$$\int_0^1 \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt = \int_0^1 \langle 2+5t, 27t^2\rangle\cdot\langle 1,3\rangle\,dt$$

$$= \int_0^1 2+5t+81t^2\,dt = \left[2t+\frac{5}{2}t^2+\frac{81}{3}t^3\right]_0^1$$

$$= 2(1)+\frac{5}{2}(1)^2+\frac{81}{3}(1)^3 = 2+\frac{5}{2}+27 = 31.5$$