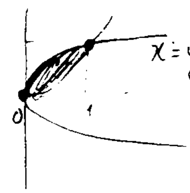
**Quiz 6 Calc IV (Math 2443-003) 7/19/2001**

> **Green's Theorem (provided the proper conditions apply):**
> $$\oint_C P\,dx + Q\,dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y) = (x^2 - y^2)\mathbf{i} - (y^2)\mathbf{j}$ where C is the positively oriented boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.



$x = y^2$ $\quad \begin{cases} y = x^2 \\ x = y^2 \end{cases}$

$$\int_C F\,dr = \iint_D \left( \underbrace{\frac{\delta(-y^2)}{\delta x}}_{=0} - \frac{\delta(x^2-y^2)}{\delta y} \right) dA$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} 2y\,dy\,dx = \int_0^1 y^2 \Big|_{x^2}^{\sqrt{x}} dx = \int_0^1 (x - x^4)\,dx$$

$$= \frac{1}{2}x^2 - \frac{1}{5}x^5 \Big|_0^1 = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

2. If $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + yz\,\mathbf{j} + xz\,\mathbf{k}$, compute curl $\mathbf{F}$.

$$\text{Curl } \vec{F} = \nabla \times \vec{F}$$

$$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix} = (0-y)\hat{i} - (z-0)\hat{j} + (0-x)\hat{k}$$

$$\boxed{\text{Curl } \vec{F} = -y\hat{i} - z\hat{j} - x\hat{k} \text{ or } \langle -y, -z, -x \rangle}$$

3. If $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + yz\,\mathbf{j} + xz\,\mathbf{k}$, compute div $\mathbf{F}$.

$$\text{div } \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle xy, yz, xz \rangle$$

$$= y + z + x \quad \text{(scalar)}$$