Problem Set 2 Calc IV (Math2443-002) Due 6/19/2002

Each problem is worth 5 points. For full credit indicate clearly how you reached your answer.

1. Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y)=3x+4y subject to the constraint $x^2+y^2=25$.

5

$\nabla f(x,y) = \lambda \nabla g(x,y)$

$\nabla f(x,y) = \langle 3, 4 \rangle$

$\nabla g(x,y) = \langle 2x, 2y \rangle$

$\langle 3,4 \rangle = \lambda \langle 2x, 2y \rangle$

$3 = \lambda 2x$

$4 = \lambda 2y$

$x^2+y^2=25$

$x = \frac{3}{2\lambda}$

$y = \frac{4}{2\lambda} = \frac{2}{\lambda}$

$\left(\frac{3}{2\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 = 25$

$\frac{9}{4\lambda^2} + \frac{4}{\lambda^2} = 25$

$\frac{9}{4\cdot 25} + \frac{4}{25} = \lambda^2$

$\lambda = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

If $\lambda = \frac{1}{2} \rightarrow (3, 4)$

If $\lambda = -\frac{1}{2} \rightarrow (-3, -4)$ $\Rightarrow f(x,y) \Rightarrow \begin{cases} 25 & \text{max } (3,4) \\ -25 & \text{min } (-3,-4) \end{cases}$

Great

2. Use Lagrange multipliers to find the maximum value of the function f(x,y)=-15cos(πx/12-π/4)+y/50+80 subject to the constraint y=60x-480 (and with 8≤t≤18). Decimal approximations are completely acceptable in this context, but must be accurate to at least two decimal places.

5

+2

$f(x,y) = -15\cos(\pi x/12 - \pi/4) + y/50 + 80$

$g(x) = 60x - 480 - y$

$f_x(x,y) = 15\sin(\pi x/12 - \pi/4)\left(\frac{\pi}{12}\right)$ $\quad f_y(x,y) = \frac{1}{50}$

$g_x(x,y) = 60$ $\quad g_y(x,y) = -1$

$\nabla f = \langle 15\sin(\frac{\pi x}{12} - \frac{\pi}{4})(\frac{\pi}{12}), \frac{1}{50} \rangle$ $\quad g_y \langle 60, -1 \rangle$

$\nabla f = \langle \frac{5\pi}{4}\sin(\frac{\pi x}{12} - \frac{\pi}{4}), 1/50 \rangle$

$f_x = \lambda g_x \Rightarrow \frac{5\pi}{4}\sin(\frac{\pi x}{12} - \frac{\pi}{4}) = \lambda 60$

$f_y = \lambda g_y \Rightarrow 1/50 = -\lambda 1 \Rightarrow \lambda = -1/50$

$f_x = -\frac{1}{50} g_x = \sin(\frac{\pi x}{12} - \frac{\pi}{4}) = -\frac{6}{5}\cdot\frac{4}{5\pi} = \frac{-24}{25\pi}$

$\sin\frac{\pi x}{12} - \frac{\pi}{4} = -\frac{24}{25\pi}$

$\sin^{-1}\left(\frac{\pi x}{12} - \frac{\pi}{4}\right) = -\frac{24}{25\pi}$

$\frac{\pi x}{12} - \frac{\pi}{4} = -0.3105$

$\frac{\pi x}{12} = -0.31 + \frac{\pi}{4}$

$x = (-0.31 + \pi/4)\cdot\frac{12}{\pi}$

$x = 1.815887223$ - min

$\max = \pi - (-0.3105...)$

$= 3.452137$

$x = (3.452137 + \pi/4)\frac{12}{\pi}$

$x = 16.186194$

for $x = 16.186194$

$y = 60(16.186194) - 480 = 491.17164$

max happens at (16.19, 491.17)

$\star$ 16.19 = 4:11 pm

at 491.17 miles south of Iowa

Excellent