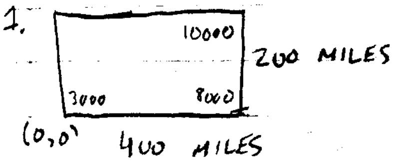
[redacted]

Problem Set 3

1.



$\delta x = \frac{8000-3000}{400} = \frac{5000}{400} = 12.5 \ \frac{\text{BUSHELS}}{\text{MILE}^3}$ ← FIND ∂x and ∂y

$\delta y = \frac{10000-8000}{200} = \frac{2000}{200} = 10 \ \frac{\text{BUSHELS}}{\text{MILE}^3}$

$f(x,y) = 12.5x + 10y + 3000$ ← Find equation for plane

$\int_0^{400}\int_0^{200} (12.5x + 10y + 3000)\,dy\,dx$ ← Double integral using state dimensions as x and y limits of integration.

$\int_0^{400} [12.5xy + 5y^2 + 3000y]_0^{200}\,dx$

5

$\int_0^{400} [12.5(200)x + 5(200)^2 + 3000(200)]\,dx$

$\int_0^{400} [2500x + \underbrace{200000 + 600000}_{800000}]\,dx$

$[1250x^2 + 800000x]_0^{400}$

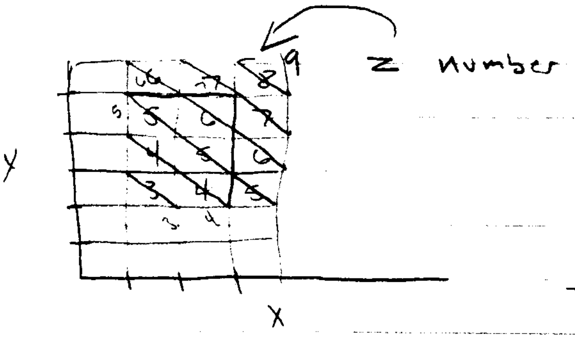
Nicely done

$1250(400)^2 + 800000(400) - 0$

520 MILLION BUSHELS ← Answer

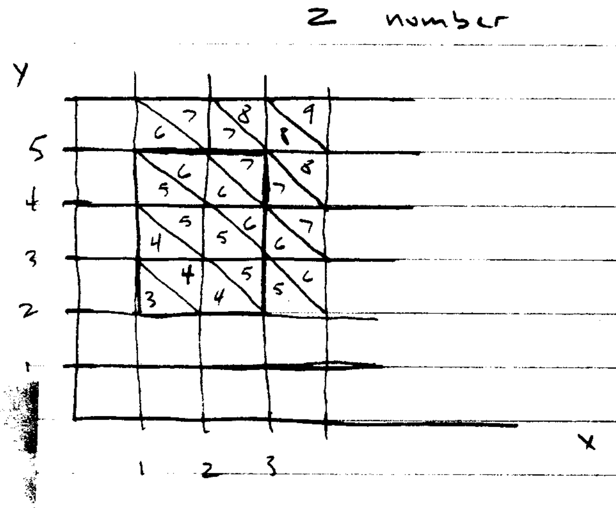
520,000,000 BUSHELS

2.



5

x	y	[x+y]
1	2	3
1	3	4
1	4	5
1	5	6
2	2	4
2	3	5
2	4	6
2	5	7
3	2	5
3	3	6
3	4	7
3	5	8
		66



Nicely done

$$\left(3\times\frac{1}{2}\right)+\left(4\times\frac{3}{2}\right)+\left(5\times 2\right)+\left(6\times\frac{3}{2}\right)+\left(7\times\frac{1}{2}\right)$$

$$\frac{3}{2}+\frac{12}{2}+10+\frac{18}{2}+\frac{7}{2}+5+15=\underline{30}$$

planes,
wedges
constant

3. cylinder $x^2+y^2=r^2$ $z=0$ $z=mx$

iterated integral for volume one

5

$$2\int_0^r\int_0^{\sqrt{r^2-x^2}} mx\,dy\,dx$$

$$\int_0^r mxy\Big]_0^{\sqrt{r^2-x^2}}\,dx$$

$$\int_0^r mx(\sqrt{r^2-x^2})\,dx$$

∴ $u=r^2-x^2$

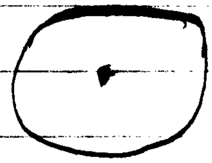
$du=-2x$

$$2\left[-\frac{m}{2}\int_0^r u^{1/2}\,dx\right.$$

Excellent

$$-\frac{m}{2}\left[\frac{2u^{3/2}}{3}\right]_0^r$$

$$2\left[\frac{-m}{2}\left[\frac{2(r^2-x^2)^{3/2}}{3}\right]_0^r\right.$$



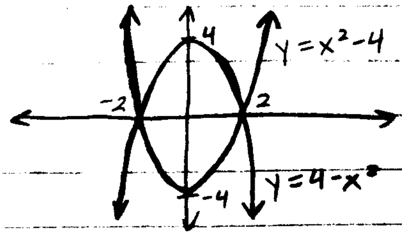
$$2\left[\frac{-m}{2}\left[\frac{2(r^2-r^2)^{3/2}}{3}-\frac{2(r^2-0)^{3/2}}{3}\right.\right.$$

$y=\pm\sqrt{r^2-x^2}$

$$2\left[\frac{-m}{2}\left[0-\frac{2(r^2)^{3/2}}{3}\right]\right.$$

$$2\left[\frac{-m}{2}\left[\frac{-2r^3}{3}\right.\right.=\boxed{\frac{2mr^3}{3}}$$

4. Top View



If the bottom boundary was 0, then $V = \int_{-2}^{2}\int_{x^2-4}^{4-x^2} (4-x^2)\,dy\,dx$.

5

Since the region below $z=0$ is symmetric to the region above $z=0$, the total volume is equal to two times the top volume: $V = 2\int_{-2}^{2}\int_{x^2-4}^{4-x^2} (4-x^2)\,dy\,dx$. The volume could also be found by solving $\int_{-2}^{2}\int_{x^2-4}^{4-x^2}\left((4-x^2)-(x^2-4)\right)dy\,dx$.

Great Job

$$V = 2\int_{-2}^{2}\int_{x^2-4}^{4-x^2} (4-x^2)\,dy\,dx$$

$$= 2\int_{-2}^{2}\left[4y - x^2y\right]_{x^2-4}^{4-x^2} dx$$

$$= 2\int_{-2}^{2}\left(4(4-x^2)-x^2(4-x^2)\right)-\left(4(x^2-4)-x^2(x^2-4)\right)dx$$

$$= 2\int_{-2}^{2}\left(16-4x^2-4x^2+x^4-4x^2+16+x^4-4x^2\right)dx$$

$$= 2\int_{-2}^{2}\left(2x^4-16x^2+32\right)dx$$

$$= 2\left[\tfrac{2}{5}x^5 - \tfrac{16}{3}x^3 + 32x\right]_{-2}^{2}$$

$$= 2\left[\left(\tfrac{64}{5} - \tfrac{128}{3} + 64\right) - \left(-\tfrac{64}{5} + \tfrac{128}{3} - 64\right)\right]$$

$$= 2\left(\tfrac{1024}{15}\right)$$

$$= \boxed{\tfrac{2048}{15}}$$