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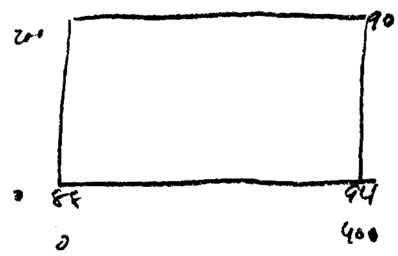
Problem Set 4

1.




$@(0,0) \to f(x,y) = 88$

$@(400,0) \to 94 = 88 + 400(a)$

$a = .015$

$@(400,200) \to 90 = 88 + .015x + 200b$

$b = -.02$

$\to f(x,y) = 88 + .015x - .02y$

$$\int_0^{400}\int_0^{200} 88 + .015x - .02y \, dy\, dx$$

$$\int_0^{400} 88y + .015xy - \frac{.02y^2}{2} \Big|_0^{200}$$

$$\int_0^{400} 17600 + 3x - 400 \, dx$$

$$17600x + \frac{3}{2}x^2 - 400x \Big|_0^{4}$$

$$7040000 + 240000 - 160000 = \underline{7120000}$$

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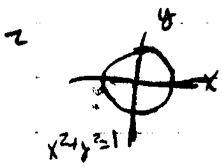
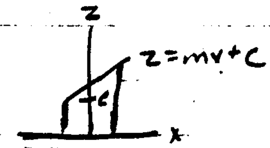
$$\text{Average} = \frac{1}{\text{Area}} \iint_A f(x,y)\, dA$$

$$\to \frac{7120000}{400 \times 200} = \boxed{89 \text{ degrees}}$$

Excellent

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$$\int_0^{2\pi}\int_0^1 mx + c \; r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 (m r\cos\theta + c)\,r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 (mr^2\cos\theta + cr)\,dr\,d\theta$$

$$\int_0^{2\pi}\left[\tfrac{1}{3}mr^3\cos\theta + \tfrac{1}{2}cr^2\right]_0^1 d\theta$$

$$\int_0^{2\pi}\left[\tfrac{1}{3}m\cos\theta + \tfrac{1}{2}c\right] d\theta$$

$$\left[\tfrac{1}{3}m\sin\theta + \tfrac{1}{2}c\theta\right]_0^{2\pi}$$

$$\tfrac{1}{3}m\sin(2\pi) + \tfrac{1}{2}c\,2\pi$$

$$\tfrac{1}{2}c\,2\pi$$

$$\underline{\pi c}$$

3. $m = \int_0^1 \int_0^{x^n} k\,dy\,dx$

$\int_0^1 ky \Big|_0^{x^n} dx$

$\int_0^1 kx^n\,dx$

$\frac{k}{n+1} x^{n+1} \Big|_0^1$

$\frac{k}{n+1}(1) = \frac{k}{n+1}$

$M_y = \int_0^1 \int_0^{x^n} kx\,dy\,dx$

$= \int_0^1 kxy \Big|_0^{x^n} dx$

$= \int_0^1 kx \cdot x^n\,dx$

$= \int_0^1 k\,x^{n+1}\,dx$

$= \frac{k}{n+2} x^{n+2} \Big|_0^1$

$= \frac{k}{n+2}(1)^{n+2} = \frac{k}{n+2}$

$M_x = \int_0^1 \int_0^{x^n} ky\,dy\,dx$

$= \int_0^1 \frac{k}{2} y^2 \Big|_0^{x^n} dx$

$= \int_0^1 \frac{k}{2} x^{2n}$

$= \frac{k}{2(2n+1)} x^{2n+1} \Big|_0^1$

$= \frac{k}{2(2n+1)}(1)^{2n+1} = \frac{k}{2(2n+1)}$

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Excellent!

$\bar{x} = \frac{M_y}{m} = \frac{\frac{\cancel{k}}{n+2}}{\frac{\cancel{k}}{n+1}} = \frac{n+1}{n+2}$

$\bar{y} = \frac{M_x}{m} = \frac{\frac{\cancel{k}}{2(2n+1)}}{\frac{\cancel{k}}{n+1}} = \frac{n+1}{2(2n+1)}$

$(\bar{x}, \bar{y}) = \left(\frac{n+1}{n+2}, \frac{n+1}{4n+2}\right)$

4. $m = \int_0^{2\pi}\int_0^{1-\sin\theta} kr\,dr\,d\theta$

$$= \int_0^{2\pi} \frac{k}{2} r^2 \Big|_0^{1-\sin\theta} d\theta$$
$$= \int_0^{2\pi} \frac{k}{2}(1-\sin\theta)^2\, d\theta$$
$$= \frac{k}{2}\cdot 3\pi = \frac{3k\pi}{2}$$

$M_x = \int_0^{2\pi}\int_0^{1-\sin\theta} kr(r\sin\theta)\,dr\,d\theta$

$$= \int_0^{2\pi} \frac{k}{3} r^3 \sin\theta \Big|_0^{1-\sin\theta} d\theta$$
$$= \int_0^{2\pi} \frac{k}{3}(1-\sin\theta)^3 \sin\theta\, d\theta$$
$$= \frac{k}{\cancel{3}}\cdot\left(-\frac{\cancel{15}\pi}{4}\right) = -\frac{5k\pi}{4}$$

$M_y = \int_0^{2\pi}\int_0^{1-\sin\theta} kr(r\cos\theta)\,dr\,d\theta$

$$= \int_0^{2\pi} \frac{k}{3} r^3 \cos\theta \Big|_0^{1-\sin\theta} d\theta$$
$$= \int_0^{2\pi} \frac{k}{3}(1-\sin\theta)^3 \cos\theta\, d\theta$$
$$= \frac{k}{3}(0) = 0$$

$$\bar{x} = \frac{M_y}{m} = \frac{0}{\frac{3k\pi}{2}} = 0 \qquad \bar{y} = \frac{M_x}{m} = \frac{-\frac{5\cancel{k\pi}}{4}}{\frac{3\cancel{k\pi}}{2}} = \frac{-10}{12} = -\frac{5}{6}$$

$$(\bar{x}, \bar{y}) = \left(0, -\frac{5}{6}\right)$$