Exam 1   Calc 1   6/24/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the definition of the derivative of the function f(x) at the point $x = a$.

$x = a$ $\qquad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

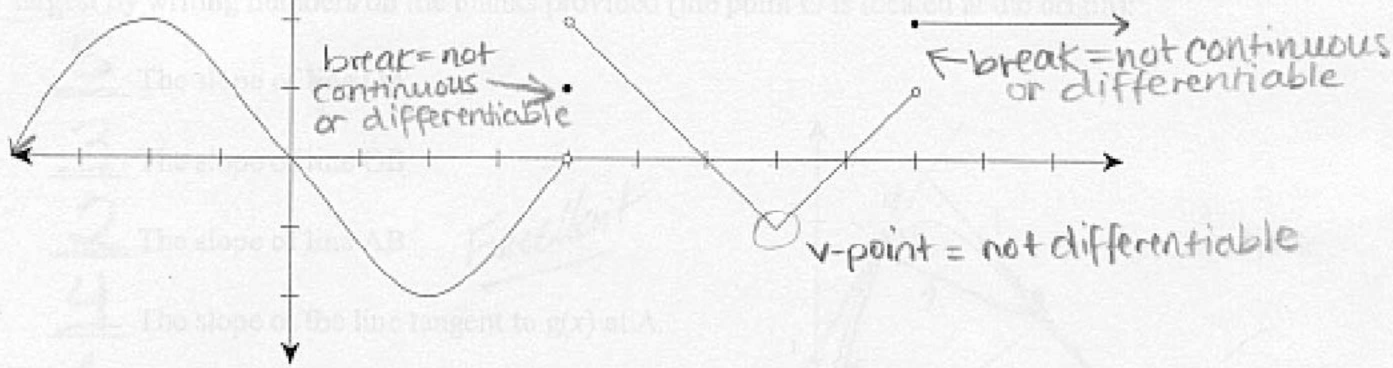
Good

2. For the function f(x) whose graph is shown below:

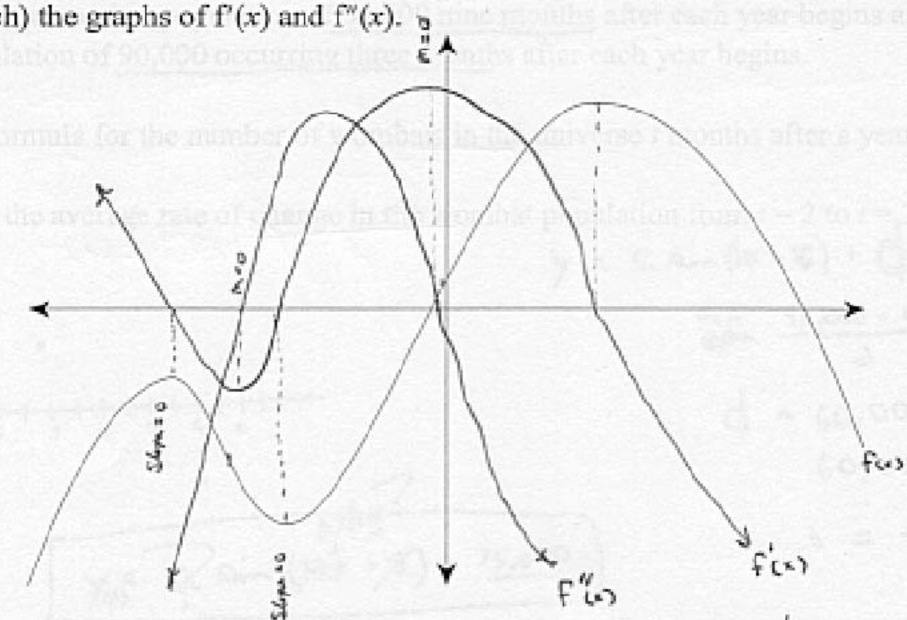
a) For which value(s) of x is f(x) not continuous? $x = 4, 9$

b) For which value(s) of x is f(x) not differentiable? $x = 4, 7, 9$

Excellent



3. Given the graph of f(x) shown below, sketch (on the same set of axes, being clear about labeling which is which) the graphs of f'(x) and f''(x).



Excellent

4. Given the graph of $g(x)$ shown below, rank the following quantities from 1 for smallest to 5 for largest by writing numbers on the blanks provided (the point O is located at the origin):

5 The slope of line OA $\frac{1.5}{.5} = 3$

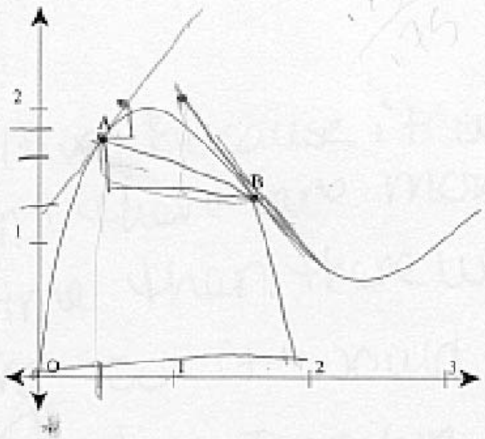
3 The slope of line OB $\frac{1.5}{2} = .75$

2 The slope of line AB $\frac{.5}{2} = .25$

4 The slope of the line tangent to $g(x)$ at A. 1

1 The slope of the line tangent to $g(x)$ at B.

Nice!



$\frac{1.25}{2} = .625$

$\frac{.5}{2}$ .25

$\frac{.75}{1}$

5. Scientists find that the population of wombats in the universe varies sinusoidally on an annual basis, with a minimum wombat population of 60,000 nine months after each year begins and a maximum wombat population of 90,000 occurring three months after each year begins.

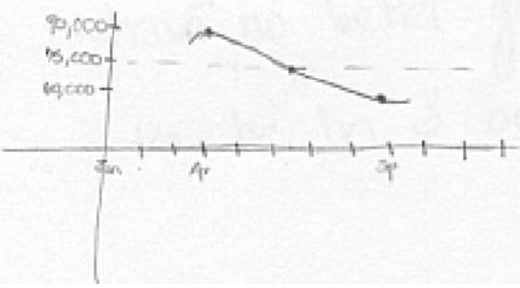
a) Find a formula for the number of wombats in the universe t months after a year begins.

$W = 15000\cos\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 75{,}000$

b) What is the average rate of change in the wombat population from $t = 2$ to $t = 2.1$?

$f'(x) = \frac{87990 - 88365}{2 - 2.1} = \frac{-375}{-.1} = \boxed{3750}$

Excellent



amp - 15,000 bats
per - 12 months
VS - 75,000 bats
PS - 3 months

$\frac{3}{12} = \frac{x}{2\pi}$

$6\pi = 12x$

$\frac{\pi}{2} = x$

6. Suppose f(t) is a function that measures the average number of cars that pass through the intersection of First avenue and 12th St. NE per hour at a time t hours after midnight.

a) What are the units on f′(t)? cars per hour per hour, or cars/hour^2

b) Would you expect the sign on f′(6.5) to be positive or negative? Why?

Positive because the number of cars passing should be increasing at 6:30 am. There should be lots more cars passing by 7am than at 6am, so the change is positive over that interval.

7. Estimate the value of $\lim_{x \to 0} \frac{3^x - 1}{x}$ correct to four decimal places. Be sure to provide justification for your reasoning.

x	f(x)
.1	1.1612
.01	1.1047
.001	1.0992
.0001	1.0986
.00001	1.0986
-.1	1.0404
-.01	1.0925
-.001	1.0980
-.0001	1.0985
-.00001	1.0986

$$\frac{1.1612 - 1.1047}{.1 - .01} = .627$$

$$\frac{1.1047 - 1.0992}{.01 - .001} = .6111$$

$$\frac{1.0992 - 1.0986}{.001 - .0001} = .6667$$

$$\frac{1.0986 - 1.0986}{.0001 - .00001} = 0$$

$$\boxed{\lim_{x \to 0} = 1.0986}$$

Well done

If you get any smaller than .00001 and get closer to 0 you will get the same answer. It happens in the negatives also.

8. Use the definition of the derivative to find the derivative function for $f(x) = \dfrac{3}{x+2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{x+h+2} - \frac{3}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3(x+2)}{(x+h+2)(x+2)} - \frac{3(x+h+2)}{(x+2)(x+h+2)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x} + \cancel{6} - \cancel{3x} - 3h - \cancel{6}}{(x+h+2)(x+2)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{(x+h+2)(x+2) \cdot \cancel{h}}$$

$$= \lim_{h \to 0} \frac{-3}{(x+h+2)(x+2)}$$

$$= \frac{-3}{(x+2)(x+2)}$$

$$= \frac{-3}{(x+2)^2}$$

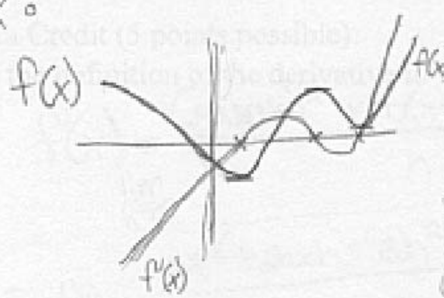
9. Biff is a calculus student at a large state university, and he's having some trouble. Biff says "Man, calculus sucks. This derivative stuff just makes no sense. The professor keeps drawing these pictures on the board and talking about stuff being positive and negative and everything, but there's like four hundred people in the class, and it's totally impossible to see the board, so I don't think I'm really getting it. So I think he's saying that a function always has to be above the axis if its increasing, is that right? I don't get why it would be, but he's said it a bunch of times I think."

Tell Biff, as clearly as possible, whether a function always has to be above the axis if its increasing. Either explain why it must be so (if you think it's true), or explain what the professor might have been actually saying (if you think Biff has misunderstood).

What the professor probably said was that if the function is increasing then the derivative will be above the axis. The derivative tells the slope of a function, so if the slope is positive, then that section on the derivative function will be positive (above the axis)

Excellent way to put it.

Ex:



The derivative tells you what's going to happen in terms of slope at a particular point. Positive derivate means that the graph of f(x) is going back up.