Exam 1 Calc 1 6/23/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the definition of the derivative of the function f(x) at the point $x = a$.

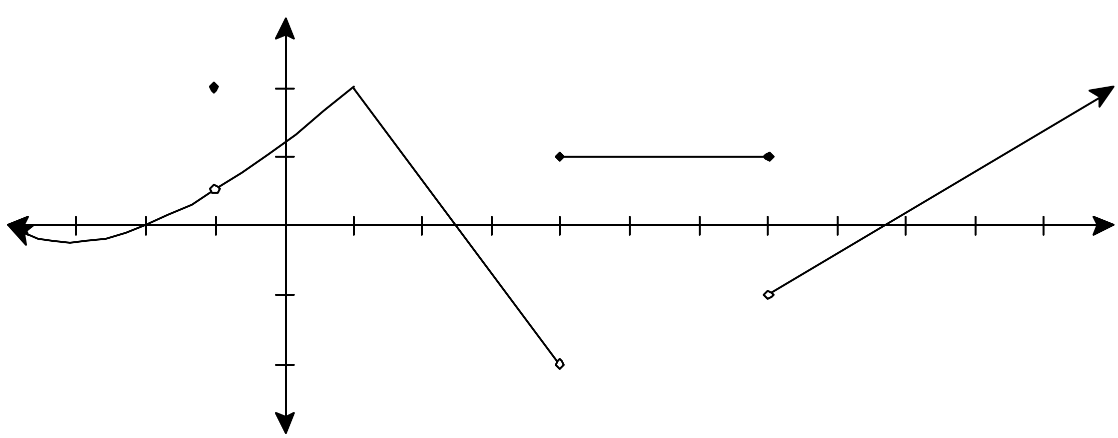
2. For the function f(x) whose graph is shown below:

   a) What is f(-1)?

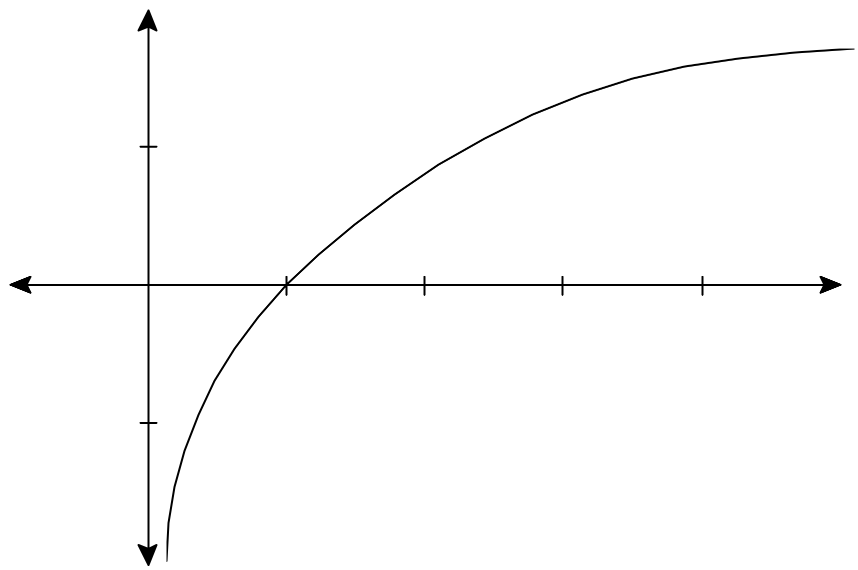
   b) What is $\lim_{x \to 4^-} f(x)$?

   c) For which value(s) of x is f(x) not continuous? Why?

   b) For which value(s) of x is f(x) not differentiable? Why?



3. Given the graph of f(x) shown below, sketch (on the same set of axes) the graph of f′(x).



4. Suppose that f(x) is a function giving the number of hours of daylight on a day x days after January 1st of 2005.

a) What are the units on f′(x)?

b) What is the sign on f′(240), and why?

5. Approximate $\lim_{x \to 0} \frac{2^x - 1}{x}$ to five decimal places.

6. If $f(t) = \dfrac{4t}{t+1}$, find $f'(t)$ using the definition of the derivative.

7. Evaluate the limit $\lim_{x \to 1}\left(3x^2 - 1\right)$ and justify each step by indicating the appropriate limit law(s) from the list below.

Algebraic Limit Properties

Let c be a constant. Then as long as $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist,

Constant Rule for Limits:	$\lim_{x \to a} c = c$
Rule X for Limits:	$\lim_{x \to a} x = a$
Sum Rule for Limits:	$\lim_{x \to a}\left[f(x) + g(x)\right] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
Difference Rule for Limits:	$\lim_{x \to a}\left[f(x) - g(x)\right] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
Constant Multiple Rule for Limits:	$\lim_{x \to a}\left[c \cdot f(x)\right] = c \cdot \lim_{x \to a} f(x)$
Product Rule for Limits:	$\lim_{x \to a}\left[f(x) \cdot g(x)\right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
Quotient Rule for Limits:	$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ as long as $\lim_{x \to a} g(x) \neq 0$.
Power Rule for Limits:	$\lim_{x \to a}\left[f(x)\right]^n = \left[\lim_{x \to a} f(x)\right]^n$

8. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, we had this test, and it was so messed up. Everything's multiple guess, of course, because there's like a million people in the class, but that means if you get something wrong you get no credit at all, and they won't tell you how to do it, they just put up the right answers afterwards so you learn *nothing*. So there was this question about, like, if you know the average rate of change of f(x) is positive on the interval from a to b, then does that mean the slope of the tangent is positive for every value from a to b. So first I thought yes, because like x^2 from 1 to 2 has a positive average rate of change, and the slope of the tangent thingy is positive all along there too, right? But then I looked at the list of answers, and one of them said the slope of the tangent thingy had to be positive for every value from a to b where it exists, and I figured out that maybe the graph could be like x^2 but with a hole in it so there wasn't any derivative there, right? So I marked that one. But they said those were wrong, and the answer was none of the above. So what's up with that? Did they get their picture backwards or something?

Tell Bunny, as clearly as possible, either what sort of situations might arise in which her conclusion fails to hold, or why her answer is in fact right so she can go argue with the professor.

9. Show **algebraically** that the function $f(x) = (x - 1)|x|$ is not differentiable at the origin. Explain your reasoning clearly.

10. Suppose $f(x)$ is a differentiable function for all values of x, and $g(x) = -3\, f(x) + 2$. What can you say about $g'(x)$?

Extra Credit (5 points possible):

For what values of a and b is $\lim_{x\to 0} \frac{\sqrt{ax+b}-2}{x} = 5$?