Exam 1 Calc 1 6/23/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the definition of the derivative of the function f(x) at the point $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Good

2. For the function f(x) whose graph is shown below:

a) What is f(-1)? 2

b) What is $\lim_{x \to 4^-} f(x)$? -2

c) For which value(s) of x is f(x) not continuous? Why?

-1, 4, 7 because there is a open circle and a closed circle at these spots indicating a jump in value

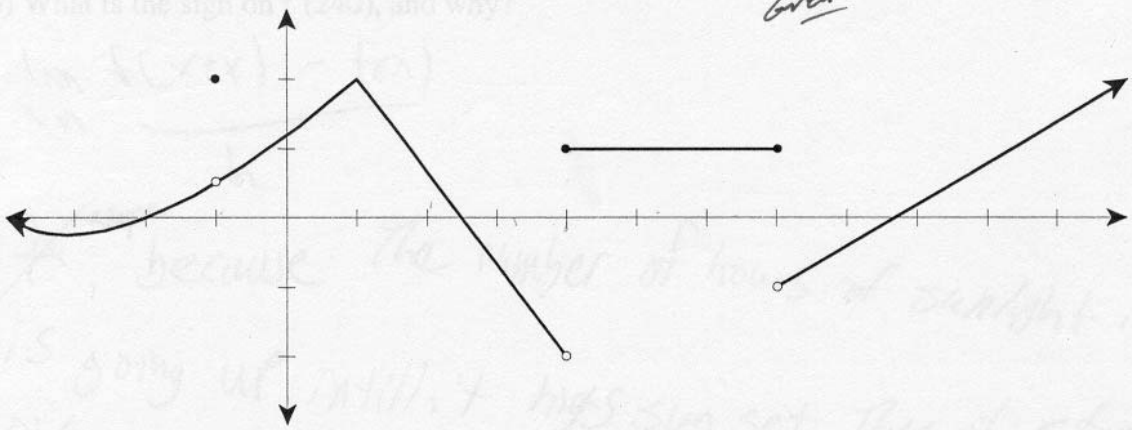
Yes

b) For which value(s) of x is f(x) not differentiable? Why?

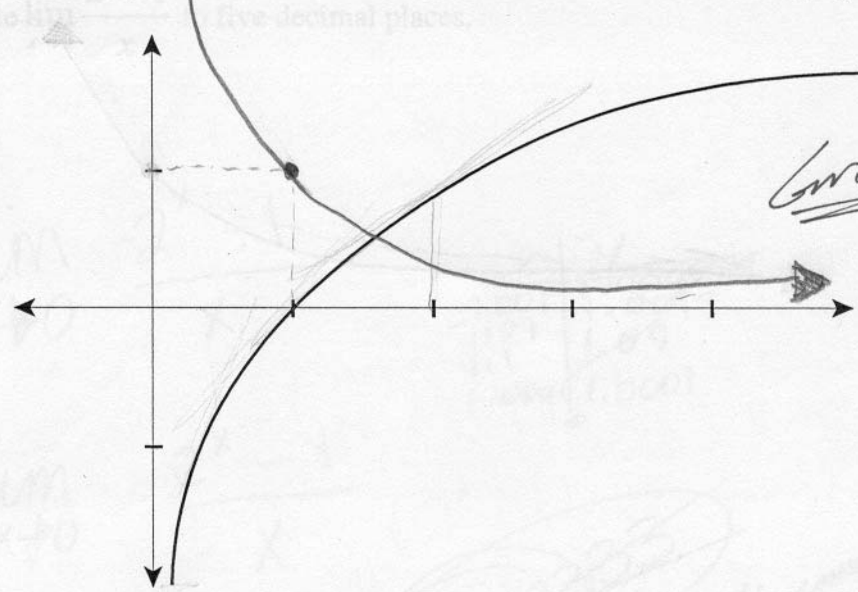
1, because it is a corner

-1, 4, 7 because at these points it is not continuous which also means that it's not differentiable

Great!



3. Given the graph of f(x) shown below, sketch (on the same set of axes) the graph of f′(x).



Great

4. Suppose that f(x) is a function giving the number of hours of daylight on a day x days after January 1[st] of 2005.

a) What are the units on f′(x)?

hours of daylight gained/lost per day $\left(\frac{\text{hours}}{\text{day}}\right)$

Yep!

b) What is the sign on f′(240), and why?

negative because it would be aproximately august and ~~to~~ the hours of daylight get less and less between June & January (might be June & December

Exactly!

5. Approximate $\lim_{x \to 0} \frac{2^x - 1}{x}$ to five decimal places.

Let's try it numerically:

x	$\frac{2^x - 1}{x}$
.1	.71773
.01	.69556
.001	.69339
.0001	.69317
.00001	.69315
.000001	.69315

Looks like it approaches

0.69315

6. If $f(t) = \dfrac{4t}{t+1}$, find $f'(t)$ using the definition of the derivative.

Use the definition:

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4(t+h)}{(t+h)+1} - \frac{4t}{t+1}}{h}$$

$$= \lim_{h \to 0} \left(\frac{4t+4h}{t+h+1} - \frac{4t}{t+1} \right) \cdot \frac{1}{h}$$

Let's get a common denominator...

$$= \lim_{h \to 0} \frac{(4t+4h)(t+1) - (4t)(t+h+1)}{(t+h+1)(t+1)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4t^2} + \cancel{4t} + \cancel{4ht} + 4h - \cancel{4t^2} - \cancel{4th} - \cancel{4t}}{h(t+h+1)(t+1)}$$

$$= \lim_{h \to 0} \frac{4\cancel{h}}{\cancel{h}(t+h+1)(t+1)}$$

$$= \lim_{h \to 0} \frac{4}{(t+h+1)(t+1)}$$

$$= \frac{4}{(t+1)(t+1)}$$

$$= \frac{4}{(t+1)^2}$$

7. Evaluate the limit $\lim_{x \to 1}\left(3x^2 - 1\right)$ and justify each step by indicating the appropriate limit law(s) from the list below.

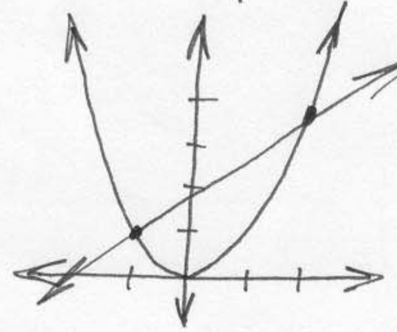
$$\lim_{x \to 1} (3x^2 - 1) \overset{\text{Diff. Rule}}{=} \lim_{x \to 1} 3x^2 - \lim_{x \to 1} 1 \overset{\text{Const. Rule}}{=} \lim_{x \to 1} 3x^2 - 1 \overset{\text{Const. Mult. Rule}}{=} 3 \lim_{x \to 1} x^2 - 1$$

$$\overset{\text{Product Rule}}{=} 3 \lim_{x \to 1} x \lim_{x \to 1} x - 1 \overset{\text{Rule X twice}}{=} 3 \cdot 1 \cdot 1 - 1 = 3 - 1 = 2$$

8. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, we had this test, and it was so messed up. Everything's multiple guess, of course, because there's like a million people in the class, but that means if you get something wrong you get no credit at all, and they won't tell you how to do it, they just put up the right answers afterwards so you learn *nothing*. So there was this question about, like, if you know the average rate of change of f(x) is positive on the interval from a to b, then does that mean the slope of the tangent is positive for every value from a to b. So first I thought yes, because like x^2 from 1 to 2 has a positive average rate of change, and the slope of the tangent thingy is positive all along there too, right? But then I looked at the list of answers, and one of them said the slope of the tangent thingy had to be positive for every value from a to b where it exists, and I figured out that maybe the graph could be like x^2 but with a hole in it so there wasn't any derivative there, right? So I marked that one. But they said those were wrong, and the answer was none of the above. So what's up with that? Did they get their picture backwards or something?

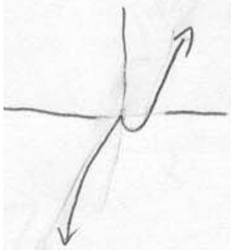
Tell Bunny, as clearly as possible, either what sort of situations might arise in which her conclusion fails to hold, or why her answer is in fact right so she can go argue with the professor.

Okay Bunny, hold on. I think you've got a good basic idea here, but you're stuck on just part of the picture. Think about the graph of $f(x) = x^2$ like you were, but about the interval from $a = -1$ to $b = 2$. Now the average rate of change is still positive, because the slope of the line connecting those two points is positive, but there are also points in between where the slope of the tangent line is negative, like where $x = -1/2$.



The key, Bunny, is that the question is a general one about what can happen with any functions. You saw one or two good possibilities, but that doesn't mean that what happens with them always happens. You have to keep thinking about whether there are other possibilities. Since just because the average rate of change is positive doesn't tell you much about what happens within the interval, in this case you can't conclude much at all.

9. Show **algebraically** that the function $f(x) = (x-1)|x|$ is not differentiable at the origin. Explain your reasoning clearly.



$$f'(0) = \lim_{h \to 0^-} \frac{f(h+0) - f(0)}{h}$$

$$= \lim_{h \to 0^-} \frac{f(h)}{h}$$

would be negative

$$= \lim_{h \to 0^-} \frac{(h-1)|h|}{h}$$

$$= \lim_{h \to 0^-} \frac{-\cancel{h}(h-1)}{\cancel{h}}$$

$$= \lim_{h \to 0^-} -h+1$$

$$= \boxed{1}$$

Well done!

$$f'(0) = \lim_{h \to 0^+} \frac{f(h+0) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(h)}{h}$$

would be pos doesn't matter

$$= \lim_{h \to 0^+} \frac{(h-1)|h|}{h}$$

$$= \lim_{h \to 0^+} \frac{\cancel{h}(h-1)}{\cancel{h}}$$

$$= \lim_{h \to 0^+} h-1$$

$$= \boxed{-1}$$

$$1 \neq -1$$

Not differentiable at $x=0$

((0,0), origin)

10. Suppose f(x) is a differentiable function for all values of x, and g(x) = -3 f(x) + 2. What can you say about g$'$(x)?

Well, algebraically:

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{-3f(x+h)+2-(-3f(x)+2)}{h}$$

$$= \lim_{h\to 0} \frac{-3f(x+h)+\cancel{2}+3f(x)-\cancel{2}}{h} = \lim_{h\to 0} \frac{-3(f(x+h)-f(x))}{h}$$

$$= -3 \cdot \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = -3 \cdot f'(x).$$

Or in terms of transformations:

The "+2" just translates vertically, so it has no impact on slope. The "$-3\cdot$" turns it upside down and stretches it away from the x axis by a factor of 3, so all the slopes have their signs reversed and are multiplied by 3.

Thus the slope of g(x) is negative 3 times the slope of f(x) at each value of x.