Exam 2 Calc 1 7/7/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State and prove the Constant Rule for derivatives.

2. Given the table of values at right, find the following:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	5	-6	2	0
1	2	1	-3	5
2	7	3	-1	17
3	2	-4	0	2

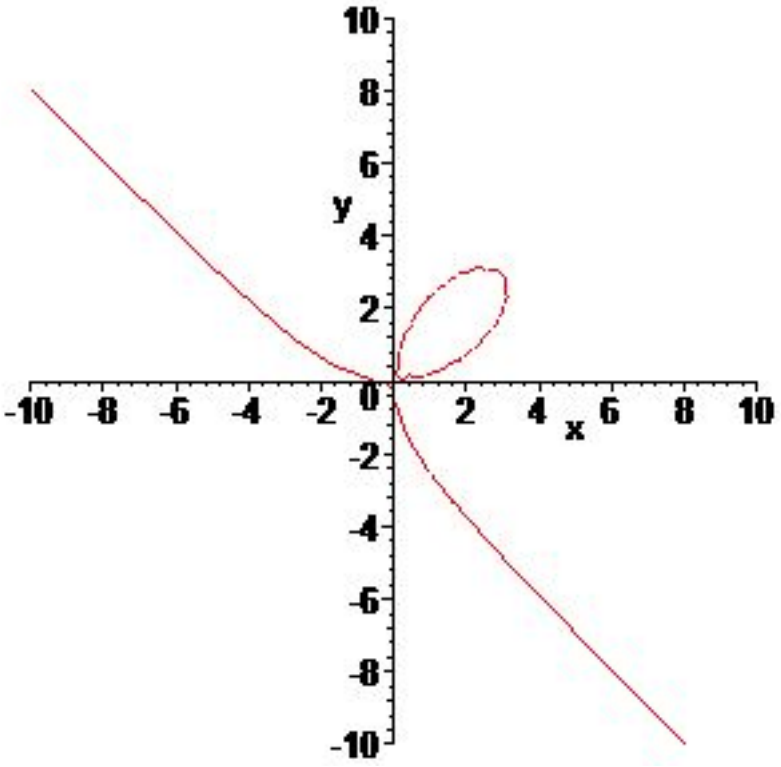
a) $F'(1)$, if $F(x) = (f / g)(x)$.

b) $G'(2)$, if $G(x) = e^{(g(x))}$.

c) $H'(3)$, if $H(x) = (f \circ g)(x)$.

3. A plane flying horizontally at an altitude of 1 mile and a speed of 600 miles per hour passes directly over Jon's house (which is just a few miles from the airport). Find the rate at which the distance from the plane to the house is increasing when it is 2 miles away from the house.

4. The curve with equation $x^3 + y^3 = 6xy$ is called the folium of Descartes. Find the equation of its tangent line at the point (3,3).



5. Show that $\left(\cos^{-1} x\right)' = \dfrac{-1}{\sqrt{1-x^2}}$.

6. State and prove the Product Rule for derivatives.

7. If $f(x) = x^{\tan x}$, find $f'(x)$.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dang, this calculus stuff is kicking my ass. I was gonna be a math major, but now I have no idea what's going on and I might switch to philosophy or something. But I still want to get this stuff. So here's what I'm trying to figure out: The T.A. said we don't really know any of this stuff unless we know how it comes from the definition of the derivative, right? And I'm good with that, because I like knowing why things are and stuff. So we learned how you prove the Product Rule and Quotient Rule and stuff from the definition, but not the Chain Rule. So does that mean we don't really know the Chain Rule agrees with the definition of the derivative?"

Help Biff out by explaining why we might have good reason to believe that the Chain Rule agrees with the definition of the derivative, even without a complete proof.

9. If $h(x) = f(x)g(x)$, where f and g are functions whose first, second, and third derivatives exist,
   a) Show that $h''(x) = f''g + 2f'g' + fg''$.

   b) Derive a formula for h''' similar to the one in part a.

10. The equation

$$y'' + 5y' - 6y = 0$$

is called a **differential equation** because it involves an unknown function y and its derivatives y' and y''. Find a constant r such that the function

$$y = e^{rx}$$

satisfies this equation.[1] [Hint: Start by finding y' and y'', then substitute them into the differential equation and see what you can do.]

Extra Credit (5 points possible):
What is special about the functions

$$f(x) = \sin^{-1}\frac{1}{\sqrt{x^2+1}} \text{ and } g(x) = \tan^{-1}\frac{1}{x}?$$

[1]Borrowed from Stewart 5th, §3.7 #55/57.