

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the maximum **value** of the function  $f(x) = -x^2 + 5x - 7$ .

$$f(x) = -1(x^2 - 5x + 7)$$

$$f'(x) = -1(2x - 5)$$

$$0 = -2x + 5$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) - 7$$

$$= -\frac{25}{4} + \frac{25}{2} - 7$$

$$= -\frac{25}{4} + \frac{50}{4} - \frac{28}{4}$$

$$= \frac{-3}{4} \text{ occurs at } x = \frac{5}{2}$$

Yes.

2. Find the **exact coordinates** of the inflection point of the function  $g(x) = x^3 + 2x^2 - 1$ .

$$g'(x) = 3x^2 + 4x$$

$$g''(x) = 6x + 4$$

$$0 = 6x + 4$$

$$-4 = 6x$$

$$x = -\frac{2}{3} \text{ is the } x \text{ value, and}$$

$$g\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^3 + 2\left(-\frac{2}{3}\right)^2 - 1 = -\frac{8}{27} + \frac{8}{9} - 1 = -\frac{8}{27} + \frac{24}{27} - \frac{27}{27}$$

$$= -\frac{11}{27} \text{ is the } y \text{ value, so}$$

$\left(-\frac{2}{3}, -\frac{11}{27}\right)$  is the actual point of inflection  
(it's easy to tell from a calculator that the graph does change concavity there.)

At an inflection point  $g''$  will be zero...