Exam 2 Calc 1 6/6/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State and prove the Constant Rule for derivatives.

2. Find an equation for the line tangent to $f(x) = x + \ln(\sin x)$ at the point where $x = \pi/2$.

3. Show that the derivative of $\cot x$ is $-\csc^2 x$.

4. Following a bank robbery, two getaway cars leave a bank at exactly noon, one heading East at 40 miles per hour and the other heading South at 60 miles per hour. Fifteen minutes later the tracking devices hidden in the cash stolen from the bank activate. How fast is the distance between the two cars changing at that incredibly exciting instant?

5. Find y' for the curve $x^3 - 3xy + y^3 = 1$.

6. Show that $\left(\sinh^{-1} x\right)' = \dfrac{1}{\sqrt{1+x^2}}$ [Feel free to use the identity $\cosh^2 x - \sinh^2 x = 1$].

7. State and prove the Quotient Rule for derivatives.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, I found this weird thing in our Calculus book. It says the derivative of ln $(4x)$ is just $1/x$, but that's screwed up because that's supposed to be the derivative of just regular ln x, right? So there can't be two different functions with the same derivative, can there? So I figure the book's wrong somehow. Do you think I'll be famous for finding it out?"

   Help Biff out by explaining either where his mistake is, or why it's okay that these two functions have the same derivative.

9. Find all points on the circle $x^2 + y^2 = 25$ whose tangent lines pass through the point (7,1).

10. If you know that $f(x)$ is a function for which $f'(x) = [f(x)]^2$, what can you say about the derivative of $f^{-1}(x)$?

Extra Credit (5 points possible): [Stewart 5th, p. 273] Show that the length of the portion of any tangent line to the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ cut off by the coordinate axes is constant.