Exam 3 Calc 1 7/20/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find a general antiderivative for the function f(x) = $\sqrt{x} + \dfrac{1}{x^2} + e^x$.

2. Find all critical number(s) of $f(x) = 4x^3 + x^4$.

3. Find the interval(s) on which $f(x) = 4x^3 + x^4$ is concave up.

4. If a company's cost to produce x units of a particular good is given by $C(x) = 3000 + 5\sqrt{x}$, what are the company's marginal cost and average cost at a production level of 10000 units?

5. Use Newton's Method with the initial approximation $x_1 = 0$ to find (to the nearest thousandth) x_3, the third approximation to a solution of the equation $e^x = 3x$.

6. 1200 square inches of material are available to construct a box with a rectangular base twice as wide as it is long and an open top. What is the largest volume such a box can have?

7. Evaluate $\lim_{x\to\infty} \frac{(\ln x)^2}{x}$.

8. Bunny is a calculus student at Enormous State University, and she's wondering about something. Bunny says "So we learned about this Newton's method thingie in Calculus, and I pretty much get it, because it's mostly just a formula, right? But the professor said something about how we should know when it *doesn't* work. Isn't that unfair? If he teaches us something that doesn't work, shouldn't he get fired or something?"

   Help Bunny out by **explaining** the circumstances under which Newton's Method fails to work, and why.

9. Suppose a river's current has speed u and a fish is swimming upstream with speed v relative to the water. The energy E expended in such a migration is

$$E(v) = \alpha \frac{v^3}{v - u}$$

where α is a positive constant. For what value of v is E minimized? [Borrowed from Blank & Kranz – and yes, migrating fish do in fact tend to swim at the predicted speed.]

10. For which values of a and b will $\lim_{x\to\infty} \frac{(\ln x)^a}{x^b} = 0$?

Extra Credit (5 points possible):

a) Wile E. Coyote has set up an explosive trap for the Roadrunner, but accidentally triggered it himself. The blast propels him straight up with an initial velocity of 50 meters per second. The acceleration due to gravity in cartoons is 5 meters per second^2. How long does it take him to peak, and how high does he go?

b) Just after peaking, the coyote switches on his Acme jetpack, which provides 10 meters per second^2 of thrust, but tragically it turns out to be strapped on backwards, propelling him straight to the ground. What is the last possible instant for him to flip over (so that the jetpack propels him upward) in order to land with a velocity of 0?