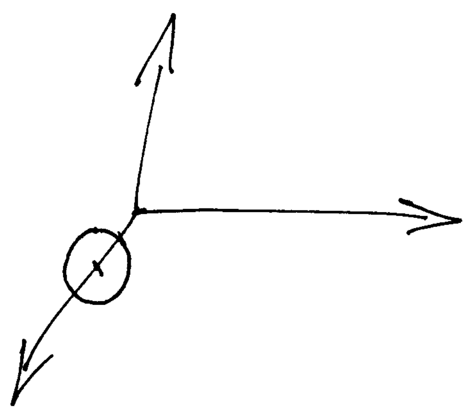
p. 902 #1

(a) $\vec{r}(t) = 2\vec{i} + \sin t\,\vec{j} + \cos t\,\vec{k}$

We can recognize this as a circle with radius 1 around the x-axis in the plane $x=2$, so it looks something like:



(b) $\vec{r}'(t) = 0\vec{i} + \cos t\,\vec{j} - \sin t\,\vec{k}$

$\vec{r}''(t) = 0\vec{i} - \sin t\,\vec{j} - \cos t\,\vec{k}$

p. 402 #6

$x = 2 - t^3 \quad y = 2t - 1 \quad z = \ln t$

(a) C intersects the xz-plane when $y = 0$, so when $0 = 2t - 1$, or $t = \frac{1}{2}$. When $t = \frac{1}{2}$ we've at the point with $x = 2 - (\frac{1}{2})^3$, $y = 2(\frac{1}{2}) - 1$, $z = \ln \frac{1}{2}$, or $(\frac{15}{8}, 0, -\ln 2)$

(b) The tangent vector is $\vec{r}'(t) = \langle -3t^2, 2, \frac{1}{t} \rangle$, so we just need to know the value of t which puts us at $(1, 1, 0)$. To find this let $1 = x = 2 - t^3$, so $t = 1$. Then $\vec{r}'(1) = \langle -3, 2, 1 \rangle$, and parametric equations for the line through $(1,1,0)$ with direction vector $\langle -3, 2, 1 \rangle$ are $x = 1 - 3t$, $y = 1 + 2t$, and $z = 0 + t$.

(c) From part (b) we know our plane's normal vector is $\langle -3, 2, 1 \rangle$, so to pass through $(1,1,0)$ the equation is: $-3(x-1) + 2(y-1) + 1(z-0) = 0$.

p. 902 #8

First compute $\frac{dx}{dt} = 3t^{1/2}$, $\frac{dy}{dt} = -2\sin 2t$, $\frac{dz}{dt} = 2\cos 2t$

Then $\text{Length} = \int_0^1 \sqrt{(3t^{1/2})^2 + (-2\sin 2t)^2 + (2\cos 2t)^2}\, dt$

$$= \int_0^1 \sqrt{9t + 4\sin^2 2t + 4\cos^2 2t}\, dt$$

$$= \int_0^1 \sqrt{9t+4}\, dt$$

let $u = 9t+4$

$\frac{du}{dt} = 9$

$\frac{du}{9} = dt$

$$= \int_{t=0}^{t=1} u^{1/2} \cdot \frac{du}{9}$$

$$= \frac{1}{9} \cdot \frac{2}{3} \cdot u^{3/2} \Big|_{t=0}^{t=1}$$

$$= \frac{2}{27} (9t+4)^{3/2} \Big|_0^1$$

$$= \frac{2}{27}(13)^{3/2} - \frac{2}{27}(4)^{3/2}$$

$$= \frac{2}{27} \cdot 13^{3/2} - \frac{16}{27}$$

p. 902 # 11

$$\vec r(t) = \left\langle \frac{t^3}{3}, \frac{t^2}{2}, t \right\rangle$$

(a) $$\vec r\,'(t) = \langle t^2, t, 1 \rangle$$

$$\vec T(t) = \frac{\vec r\,'(t)}{|\vec r\,'(t)|} = \frac{1}{\sqrt{t^4+t^2+1}} \langle t^2, t, 1 \rangle$$

(b) We didn't talk about the unit normal vector in class, but the formula is on p. 888. First find $\vec T\,'(t)$, then divide by its length.

$$\vec T\,'(t) = -\frac{1}{2}(t^4+t^2+1)^{-3/2} \cdot (4t^3+2t)\langle t^2, t, 1\rangle + \frac{1}{\sqrt{t^4+t^2+1}}\langle 2t, 1, 0\rangle$$

$$= \frac{-2t^3-t}{(t^4+t^2+1)^{3/2}}\langle t^2, t, 1\rangle + \frac{1}{\sqrt{t^4+t^2+1}}\langle 2t, 1, 0\rangle$$

$$= \left\langle \frac{-2t^5-t^3}{(t^4+t^2+1)^{3/2}}, \frac{-2t^4-t^2}{(t^4+t^2+1)^{3/2}}, \frac{-2t^3-t}{(t^4+t^2+1)^{3/2}} \right\rangle + \left\langle \frac{2t^5+2t^3+t}{(t^4+t^2+1)^{3/2}}, \frac{t^4+t^2+1}{(t^4+t^2+1)^{3/2}}, \frac{0}{(t^4+t^2+1)^{3/2}} \right\rangle$$

$$= \left\langle \frac{2t}{(t^4+t^2+1)^{3/2}}, \frac{-t^4+1}{(t^4+t^2+1)^{3/2}}, \frac{-2t^3-t}{(t^4+t^2+1)^{3/2}} \right\rangle$$

so $$|\vec T\,'(t)| = \sqrt{\frac{4t^2}{(t^4+t^2+1)^3} + \frac{(1-t^4)^2}{(t^4+t^2+1)^3} + \frac{(-2t^3-t)^2}{(t^4+t^2+1)^3}} = \frac{\sqrt{t^8+4t^6+2t^4+5t^2}}{(t^4+t^2+1)^{3/2}}$$

and $$\vec N(t) = \frac{\langle 2t, 1-t^4, -2t^3-t\rangle}{\sqrt{t^8+4t^6+2t^4+5t^2}}$$

(c) The good news after all that crap in (b) is we can use it as an easy way to find curvature, because $\kappa(t) = |\vec T\,'(t)|/|\vec r\,'(t)|$, so

$$\kappa(t) = \frac{\sqrt{t^8+4t^6+2t^4+5t^2}}{(t^4+t^2+1)^2}.$$

p. 903 #19

Put the origin 7 ft. beneath the point of release and use $\vec{i}$ for the horizontal direction and $\vec{j}$ for the vertical, so $\vec{r}(0) = 0\vec{i} + 7\vec{j}$. The starting velocity is at a $45°$ angle, so in the direction of $\vec{i} + \vec{j}$, and so we want $\vec{v}(0) = k(\vec{i} + \vec{j})$ with a value of k for which $|\vec{v}(0)| = 43$, so $43 = \sqrt{k^2 + k^2}$ and $k = \frac{43}{\sqrt{2}}$, so $\vec{v}(0) = \frac{43}{\sqrt{2}}\vec{i} + \frac{43}{\sqrt{2}}\vec{j}$. The only acceleration is due to gravity, so $\vec{a}(t) = 0\vec{i} - 32\vec{j}$.

Now since $\vec{a}(t) = 0\vec{i} - 32\vec{j}$,

$\vec{v}(t) = (0 + C_1)\vec{i} + (-32t + C_2)\vec{j}$, and using $\vec{v}(0) = \frac{43}{\sqrt{2}}\vec{i} + \frac{43}{\sqrt{2}}\vec{j}$ this is

$\vec{v}(t) = \frac{43}{\sqrt{2}}\vec{i} + \left(-32t + \frac{43}{\sqrt{2}}\right)\vec{j}$, so

$\vec{r}(t) = \left(\frac{43}{\sqrt{2}}t + C_3\right)\vec{i} + \left(-16t^2 + \frac{43}{\sqrt{2}}t + C_4\right)\vec{j}$, and since $\vec{r}(0) = 0\vec{i} + 7\vec{j}$:

$$\vec{r}(t) = \frac{43}{\sqrt{2}}t\,\vec{i} + \left(-16t^2 + \frac{43}{\sqrt{2}}t + 7\right)\vec{j}.$$

(a) When $t = 2$,

$$\vec{r}(2) = \frac{43}{\sqrt{2}}(2)\vec{i} + \left(-16(2)^2 + \frac{43}{\sqrt{2}}(2) + 7\right)\vec{j}$$

$$= \frac{86}{\sqrt{2}}\vec{i} + \left(-64 + \frac{86}{\sqrt{2}} + 7\right)\vec{j}$$

$\approx 60.8\vec{i} + 3.8\vec{j}$, so about 60 feet downfield and 4 feet above ground.

(b) The peak will be when the $\vec{j}$ component of $\vec{v}(t)$ is 0, so when:

$-32t + \frac{43}{\sqrt{2}} = 0$, or $t = \frac{+43}{32\sqrt{2}} \approx .95$, and

$\vec{r}(.95) \approx 28.9\vec{i} + 21.4\vec{j}$, so the maximum height is a little over 21 feet.

(c) It lands when the height is 0, so when:

$-16t^2 + \frac{43}{\sqrt{2}}t + 7 = 0$, and the quadratic formula gives $t \approx 2.11$.

$\vec{r}(2.11) \approx 64.2\vec{i}$, so the shot lands about 64 feet away.