

Exam 2 Calculus 1 10/11/2002

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Past returns do not assure future performance.

1. Compute the derivative of $f(x) = x \ln x - x$.

2. A table of values for f , g , f' , and g' appears at right.

(a) If $h(x) = f(g(x))$, find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	3	5	1
3	7	2	8	0

(b) If $H(x) = \frac{f(x)}{g(x)}$, find $H'(2)$.

3. Write an **equation** for the line tangent to $f(x) = e^x$ when $x=1$.

4. For the implicitly defined ellipse $5x^2 - 6xy + 5y^2 = 16$,
a) Find y' .

b) Find the **slope** of the line tangent to the ellipse at the point $(1,-1)$.

5. Jeb is driving north toward an intersection with a load of manure in the back of his truck at a rate of 60 miles per hour. Zeke is driving west toward the same intersection in a tanker truck full of toxic waste at a rate of 80 miles per hour. Obviously they're going to collide spectacularly, but putting that aside for the moment: At the instant when Jeb is 3 miles south of the intersection and Zeke is 4 miles east of the intersection, how fast are they approaching each other?

6. Show that the derivative of $f(x) = \csc x$ is $f'(x) = -\csc x \cot x$ (Remember that $\csc x = 1/\sin x$).

7. Samantha is a calculus student at Enormous State University, and she's having some trouble with derivatives of trig functions. Samantha says "So, like, for that problem on the test about the derivative of cotangent, that was easy, right? I know cotangent is $\cos x$ over $\sin x$, and since the derivative of $\cos x$ is $-\sin x$ and the derivative of $\sin x$ is $\cos x$, then the derivative of $\cos x$ over $\sin x$ is $-\sin x$ over $\cos x$. That means the derivative of $\cot x$ is $-\tan x$."

Then Samantha's friend Chuck says "But here in my notes from class it says that the derivative of $\cot x$ is $-\csc^2 x$."

Critique Samantha's derivative. If it's right, explain why the class notes would say something different; if it's wrong, explain the mistake

Help Samantha by explaining, in terms she can understand, either how to convince her professor she's right, or how it is that terms less than 1 can have a limit equal to 1.

8. Prove the quotient rule from the definition of the derivative (you may use the product rule as well if you find it convenient).

9. Show that the derivative of $y = \sin^{-1}(x)$ is $y' = \frac{1}{\sqrt{1-x^2}}$.

10. Let $f(x) = \sin 2x$. For what values of x does the line tangent to f have a slope of 1?

Extra Credit (5 points possible):

Find a version of the product rule which tells how to do the derivative of a product of three functions, i.e. a formula for $[f(x) \cdot g(x) \cdot h(x)]'$