Exam 4 Calculus 1 12/4/2002

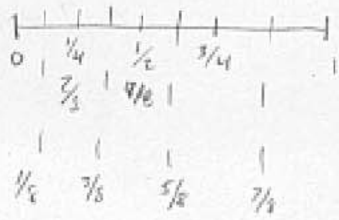
Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Don't forget the +C.

1. Estimate the area under $f(x) = \sin\sqrt{x}$ between x = 0 and x = 1 using 4 rectangles and midpoints.

$\Delta x = \frac{1-0}{4} = \frac{1}{4}$

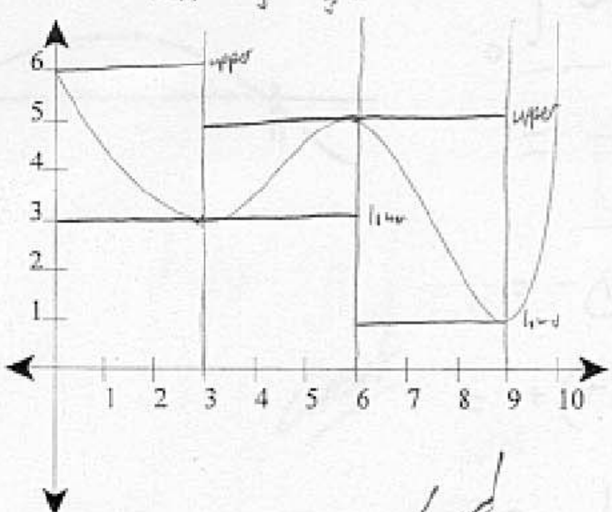
$f(\frac{1}{8})\cdot\frac{1}{4} + f(\frac{3}{8})\cdot\frac{1}{4} + f(\frac{5}{8})\cdot\frac{1}{4} + f(\frac{7}{8})\cdot\frac{1}{4}$

$\frac{1}{4}\left(\sin\sqrt{\frac{1}{8}} + \sin\sqrt{\frac{3}{8}} + \sin\sqrt{\frac{5}{8}} + \sin\sqrt{\frac{7}{8}}\right)$



2. By reading values from the graph of f(x) below, use three rectangles to find an upper estimate and a lower estimate for the area under the graph of f(x) but above the x axis between x=0 and x=9.

$\Delta x = \frac{9-0}{3} = \frac{9}{3} = 3$



Upper

$f(0)\cdot 3 + f(6)\cdot 3 + f(6)\cdot 3$

$6\cdot 3 + 5\cdot 3 + 5\cdot 3$

$18 + 15 + 15$

$= 48$

Lower

$f(3)\cdot 3 + f(3)\cdot 3 + f(9)\cdot 3$

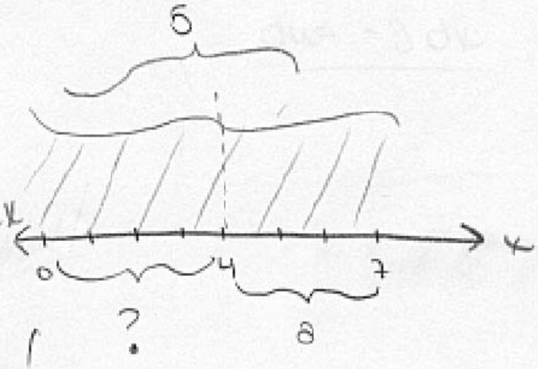
$3\cdot 3 + 3\cdot 3 + 1\cdot 3$

$9 + 9 + 3$

$= 21$

Good

3. If $\int_0^7 f(x)\,dx = 5$ and $\int_4^7 f(x)\,dx = 2$, what is $\int_0^4 f(x)\,dx$?



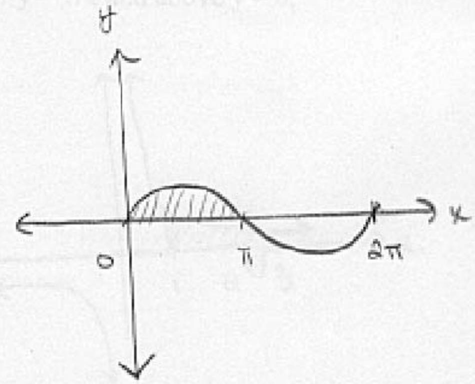
$\int_0^4 f(x)dx = \int_0^7 f(x)dx - \int_4^7 f(x)dx$

W $\int_0^4 f(x)dx = 5 - 2$

$\int_0^4 f(x)dx = 3$

Yes!

4. Find the area underneath the function f(x) = sin x but above the x axis between x=0 and x=π.



$A = \int_0^{\pi} \sin x \, dx$

W

$A = [-\cos x]_0^{\pi}$

$A = [-\cos \pi] - [-\cos 0]$

$A = 1 + +1$

$A = 2$

Great!

5. Evaluate $\int \frac{1}{5-3x}dx$. $= \int \frac{1}{u}\frac{du}{-3} = -\frac{1}{3}\int \frac{1}{u}du$

$u = 5-3x$

$\frac{du}{dx} = -3$

$du = -3dx$

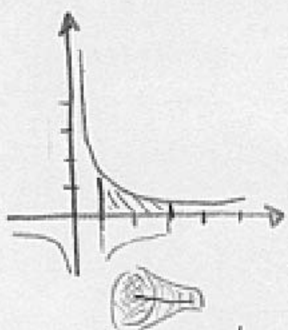
$dx = \frac{du}{-3}$

$-\frac{1}{3}\cdot \ln|u|$

$-\frac{1}{3}\cdot \ln|5-3x| + C$

Excellent

6. Find the volume of the solid generated by rotating the region under y = 1/x and above y = 0, between x = 1 and x = 3, around the x axis.



πr^2

$\pi \int_1^3 \left(\frac{1}{x}\right)^2 dx$

$\pi \int_1^3 \left(\frac{1}{x}\right)\left(\frac{1}{x}\right) dx$

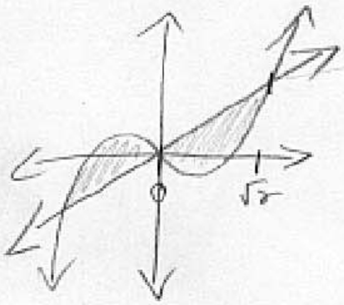
$\pi \int_1^3 \frac{1}{x^2}\, dx$

$\pi \int_1^3 x^{-2} \to -x^{-1} \to \pi - \frac{1}{x}\Big|_1^3$

$\pi\left(\frac{1}{-(3)} - \frac{1}{-(1)}\right) = \pi\left(-\frac{1}{3}+1\right) = 2/3\pi$

Well done!

7. Find the exact area of the region bounded between $f(x) = x^3 - x$ and $g(x) = x$.



$$x^3 - x = x$$
$$x^3 - 2x = 0$$
$$x(x^2 - 2) = 0$$
$$x = \sqrt{2}$$

$$2\left(\int_0^{\sqrt{2}} x\,dx - \int_0^{\sqrt{2}} x^3 - x\right)$$

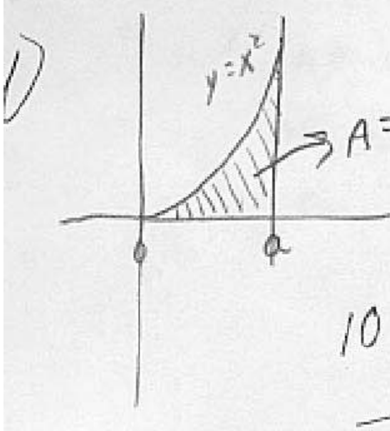
Good job

$$2\left(\left(\tfrac{1}{2}x^2\right)\Big|_0^{\sqrt{2}} - \left(\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right)\Big|_0^{\sqrt{2}}\right)$$

$$= 2\left[\left(\tfrac{1}{2}(\sqrt{2})^2 - 0\right) - \left(\left(\tfrac{1}{4}\sqrt{2}^4 - \tfrac{1}{2}\sqrt{2}^2\right) - 0\right)\right]$$

$$= 2\left[(1 - 0) - (1 - 1 - 0)\right] = 2(1 - 0) = 2$$

8. Jon wants to have a huge mathematical sculpture built in the center of the quad. It's going to be shaped like the region under $y = x^2$, and of course above $y = 0$, between $x = 0$ and $x = b$, but Jon can't decide how big b should be. If he can only afford 10 square meters of steel plate to build his sculpture with, how big should he make it?



$? = a$

Excellent!

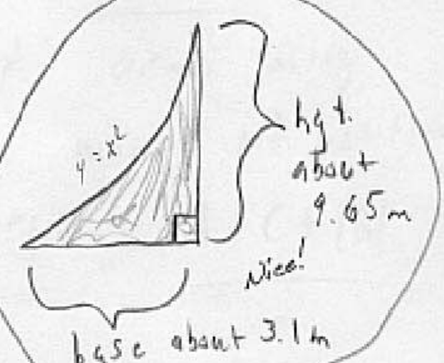
$$10 = \int_0^a x^2\,dx$$

$$10 = \left[\frac{x^3}{3}\right]_0^a$$

$$\frac{a^3}{3} - \frac{\cancel{0}^3}{3} = 10$$

$$\frac{a^3}{3} = 10$$

$$a^3 = 30$$
$$a = \sqrt[3]{30}$$
$$a \approx 3.10$$

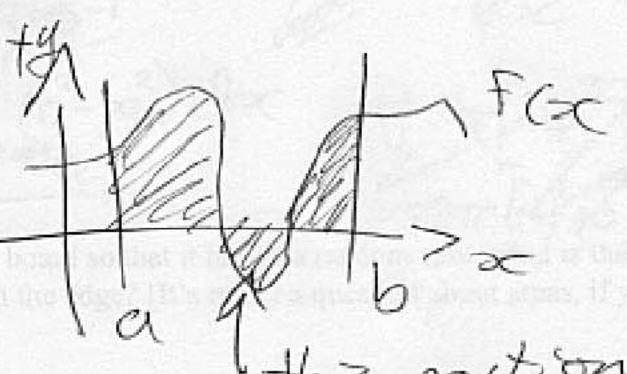


9. The first edition of the Cliff'sNotes Calculus Quick Review says on page 116 that the area A of the region bounded by the graph of f(x), the x axis, and the lines x = a and x = b is given by $A = \left| \int_a^b f(x)\,dx \right|$ when the function f(x) is sometimes above and sometimes below the x axis.

Explain in at least a few sentences why the absolute value bars in this expression do or don't accurately express the actual area bounded by these curves.

This would not give you the area all the time with the absolute value bars outside the integral like that. The integral would give you the sum of the areas between the function and the x axis, if somewhere, between a and b, the function drops bellow the x axis, the integral would find a negative number, which would be added onto the total.

Like this



Yes!

this portion negative would be subtracted from total integral from a to b

What you need to find the actual area from a to ~~Be~~ b is

$$A = \int_a^b |f(x)|\,dx$$

Exactly!

$y^2 = r^2 - x^2$

10. A sphere can be obtained by taking the top half of a disc $x^2 + y^2 = r^2$ and rotating it around the x axis. Show that the volume of this sphere is $\frac{4\pi r^3}{3}$. [If it's too confusing with the r there, try it with a 1 in that spot to warm up.]

$y = \pm\sqrt{r^2 - x^2}$

$$2\pi \int_0^r \left(\sqrt{r^2-x^2}\right)^2 dx = 2\pi \int_0^r r^2 - x^2\, dx$$

$$= 2\pi \left[r^2 x - \frac{x^3}{3} \right]_0^r \quad 2\pi \left[r^2 r - \frac{r^3}{3} \right] - 2\pi [0]$$

10

$$2\pi \left[r^3 - \frac{r^3}{3} \right] = 2\pi \left[\frac{3r^3}{3} - \frac{r^3}{3} \right] = 2\pi \left(\frac{2r^3}{3} \right) = \boxed{\frac{4\pi r^3}{3}}$$

Nicely done!