Exam 1 Calculus 3 9/18/2002

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. All warrantees void in case of improper use.

1. Write the first four terms of the sequence $\left\{ \frac{n-1}{n^2} \right\}$.

2. Write the first four partial sums of the series $\sum_{n=1}^{\infty} \frac{n-1}{n^2}$.

3. Give an example of a series which is convergent but not absolutely convergent.

4. Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5+n}}$ converges or diverges.

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges or diverges.

6. Determine whether the series $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges or diverges.

7. Write the third degree Taylor polynomial for the function f(x)=ln x centered at a=2.

8. Chaz is a calculus student at Enormous State University, and he's having trouble with series. Chaz says "Ya know, I used to be pretty good at math, but this series crap is just outta control. What's up with this thing where you do a bunch of work, and it turns out it's no good? Like with that ratio test thing, you know? You do it, and you get 1, and they say that means you have to try something else. It's like it's just a conspiracy or something, because I did it for that one over n squared series, and it was like a total waste of time, and the series converges anyway. So why the heck don't they just say if you get 1 from the ratio test, it's gonna converge?"

Explain to Chaz, in terms he can understand, whether a 1 from the ratio test means that a series converges or not.

9.Find the radius of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n^2}$.

10. Use the 6th degree polynomial for $\sin(x^2)$ to approximate $\int_0^1 \sin(x^2)\,dx$.

Extra Credit (This problem can replace your lowest other problem on the test): Find the sum of the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8} + \frac{1}{9} + \frac{1}{16} + \frac{1}{27} + \frac{1}{32} + \frac{1}{64} + \frac{1}{81} + \ldots$, where the terms are of the form 1 over all possible natural number exponents on 2 or 3.