Exam 2 Calculus 3 10/18/2002

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Watch that first step.

1. Write an equation for the line between the points (-3,2,5) and (1,-2,4).

2. Convert the point $(1,-1,\sqrt{2})$ from rectangular to
(a) cylindrical coordinates.

(b) spherical coordinates.

3. Transform the equation $x^2 + z^2 + 4 = y + 4z$ to standard form, identify the quadric surface it represents, give the coordinates of any vertex or vertices, and visualize it shaded in lavender.

4. Show that the length of the conic spiral $\mathbf{r}(t) = \langle t \cos t, t \sin t, t \rangle$ between the points (0,0,0) and $(4\pi, 0, 4\pi)$ is given by $\int_0^{4\pi} \sqrt{2 + t^2}\, dt$.

5. Wile E. Coyote leaps off a ledge as the road runner zips past below him on a perfectly level road. Wile's Acme jetpack propels him in the direction of the positive y axis with an acceleration of of 20m/s^2, and gravity in cartoons is 5m/s^2 in the direction of the negative z axis, so his acceleration is given by the vector $\langle 0, 20, -5\rangle$. Wile's jump was from 10 meters above the origin with an initial velocity of 2m/s along the positive x axis, so his initial velocity is given by the vector $\langle 2, 0, 0\rangle$ and his initial position by the vector $\langle 0, 0, 10\rangle$. What is Wile's position after two seconds?

6. Show that for any vectors **a** and **b**, the vector **b** is perpendicular to $\mathbf{a}\times\mathbf{b}$.

7. Muffy is a calculus student at E.S.U., and she's having trouble with lines in $\mathbb{R}^3$. Muffy says "I, like, totally can't get any of the answers in the back of the book. I can, like, figure out what the little vector-thingies are, and they say, like, what points to have on the line, but then when I put it together it's not what the solutions manual says. My Daddy hired a tutor to help me and he says I did everything okay, but I don't think that can be right because it's not what's in the book. I mean, like, it's math, so there's always a right answer, right?"

Explain to Muffy, in terms she can understand, **at least three** factors that could lead to correct equations for the same line which still don't look the same.

8. Find an equation for the plane containing the line $x = 2 + t$, $y = 3 - t$, $z = 4 - 2t$ and passing through the point $(5, -2, 3)$.

9. Show that a circle with a radius of 3 has a curvature of 1/3.

10. On the problem set we saw that the set of points equidistant from two fixed points forms a plane. Find the set of points whose distance from (0,0,0) is equal to twice their distance from (3,0,0).

Extra Credit (up to 5 points possible): Given the lines represented by $\mathbf{r}_1(t)=\langle 1+t, t-3, -2t\rangle$ and $\mathbf{r}_2(t)=\langle 5-3t, 3t, 7\rangle$, find a vector equation for the line which intersects both of these lines and is perpendicular to each of them.