Exam 3 Calculus 3 10/25/2002

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. It takes two to tango.

1. State the definition of the partial derivative with respect to y of a function f(x,y).

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ does not exist.

3. Let $f(x,y) = 1 + 2x\sqrt{y}$. Find the directional derivative of f at the point (3,4) in the direction of the vector **v** = <4,-3>.

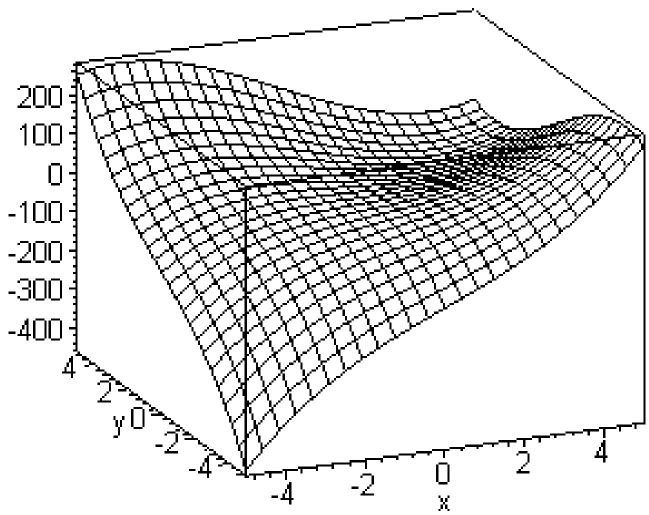
4. Find an equation of the tangent plane to the surface $z = y^2 - x^2$ at the point (-4,5,9).

5. If $z = f(x,y)$, $x = x(s,t)$, and $y = y(s,t)$, write the appropriate version of the chain rule for $\dfrac{\partial z}{\partial s}$.

Be careful to indicate clearly in your answer which derivatives are partials.

6. Find the maximum rate of change of the function $f(x,y) = e^{y-x}$ at the point (2,-3) and the direction in which it occurs.

7. The function $f(x,y) = x^3 + y^3 - 9xy + 27$ has critical points at (0,0) and (3,3). Classify these two critical points. The graph provided can serve as a guide, but it's up to you to demonstrate things.



8. Chaz is a calculus student at E.S.U., and he's having trouble with derivatives of functions of more than one variable. Chaz says "Man, this just makes no sense. I mean, I totally got it in calc 1, because slopes were just numbers and I can handle that, you know? So if the derivative was positive, it was going up, and negative meant it was going down, totally clear. But now there's, like, x part derivatives and y part derivatives, and that's just too strange. I mean, either it's going up or it's going down, you know? So how could it be, like, both going up if the x part is positive and going down 'cause the y part is negative? It can't be doing both, so how can there be different part derivatives?"

Clarify for Chaz, in terms he can understand, how to think about derivatives of functions of more than one variable, and how at a particular point different derivatives could be both positive and negative.

9. Show that the plane tangent to the surface $f(x,y) = x^2 - y^2$ at the point (x_0,y_0,z_0) has $-z_0$ as its z-intercept.

10. Consider the paraboloid $f(x,y) = x^2 + y^2$ and the collection of paraboloids $g(x,y) = -(x-1)^2 - (y-2)^2 + c$ for various values of the constant c. Exactly one of these g's should be tangent to f. For which value of the constant c will g and f be tangent?

Extra Credit (up to 5 points possible):
Prove the formula $\text{grad}(f \cdot g) = f \cdot \text{grad } g + g \cdot \text{grad } f$, where f and g are differentiable functions of the two variables x and y..