Problem Set 5 Calc 3 Due 11/8/2002

[5pts.] 1. Suppose that in some year the average wheat harvest in the state of Kansas is 3000 bushels per square mile at the southwest corner, 8000 bushels per square mile at the southeast corner, and 10000 bushels per square mile at the northeast corner. You may also assume that the wheat production varies linearly, and that Kansas is close enough to being a rectangle 200 miles from south to north and 400 miles from west to east. Set up and use a double integral to compute the total wheat production of Kansas.

[5pts.] 2. Suppose that on some day the temperature at 3pm in the state of Kansas is 88 degrees at the southwest corner, 94 degrees at the southeast corner, and 90 degrees at the northeast corner. You may also assume that the temperature varies linearly, and that Kansas is close enough to being a rectangle 200 miles from south to north and 400 miles from west to east. Set up and use a double integral to compute the average temperature in Kansas.

[5pts.] 3. If [x] denotes the greatest integer less than or equal to x, evaluate the integral $\iint_R [x+y]\,dA$ where $R = \{(x,y) | 1 \le x \le 3,\ 2 \le y \le 5\}$.

[5pts.] 4. Pat the mathematician runs a catering business during summer break. Pat is making a deli tray, and begins to ponder the volumes of irregular slices of sausage. Suppose the sausage is shaped like the cylinder $x^2+y^2=1$, and slice out a small wedge by cutting it along the planes $z=0$ and $z=mx$ (for some constant m). Set up and evaluate an iterated integral for the volume of one of the wedges that results.

[5pts.] 5. Pat the mathematician runs a catering business during summer break. Pat is making a deli tray, and begins to ponder the volumes of irregular slices of sausage. Suppose the sausage is shaped like the cylinder $x^2+y^2=1$, with one cut made perpendicular to the cylinder along the plane $z=0$ and the other slice made along the plane $z=mx+c$ (for some positive constants c and m, with $c>m$). Set up and evaluate an iterated integral for the volume of one of the wedges that results.

[5pts.] 4. Set up an iterated integral and evaluate it to find the volume of the solid bounded by the surfaces $z=4-x^2$, $z=x^2-4$, $y=4-x^2$, and $y=x^2-4$.