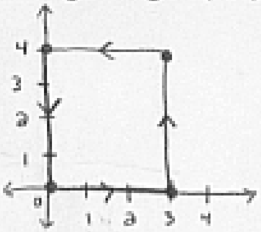
# Quiz 5  Calc 3  Due 12/3/2002

1. Compute $\int_C \mathbf{F}\cdot d\,\mathbf{r}$ for the vector field $\mathbf{F}(x,y)=(x^2y - 1)\mathbf{i} - (y^2)\mathbf{j}$ where C is the rectangular path beginning at (0,0), proceeding to (3,0), then (3,4), then (0,4), and finally back to (0,0).



$$F(x,y) = \langle \overset{P}{x^2y-1},\ \overset{Q}{y^2} \rangle$$

$$\oint_c P\,dx + Q\,dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$\int_0^4 \int_0^3 (0 - x^2)\,dx\,dy$$

$$\int_0^4 \left[-\frac{x^3}{3}\right]_0^3 dy \rightarrow \int_0^4 [-9-0]\,dy$$

$$[-9y]_0^4 \rightarrow [-36-0] = \boxed{-36}$$

This is a simple closed curve, so use Green's Theorem. Yes

Excellent

2. If $\mathbf{F}(x,y,z) = x^2y\,\mathbf{i} - 8z\,\mathbf{j} + x^3yz\,\mathbf{k}$, compute curl $\mathbf{F}$.

$$F(x,y,z) = \langle x^2y, -8z, x^3yz \rangle \qquad \text{curl } \vec{F} = \nabla \times \vec{F}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & -8z & x^3yz \end{vmatrix} = \left[\frac{\partial}{\partial y}(x^3yz) - \frac{\partial}{\partial z}(-8z)\right]\vec{i} + \left[\frac{\partial}{\partial z}(x^2y) - \frac{\partial}{\partial x}(x^3yz)\right]\vec{j} +$$

$$\left[\frac{\partial}{\partial x}(-8z) - \frac{\partial}{\partial y}(x^2y)\right]\vec{k} =$$

$$(x^3z + 8)\vec{i} + (0 - 3x^2yz)\vec{j} + (0 - x^2)\vec{k} =$$

$$\boxed{(x^3z+8)\vec{i} - 3x^2yz\,\vec{j} - x^2\vec{k}} = \langle x^3z+8, -3x^2yz, -x^2 \rangle$$

Good

3. If $\mathbf{F}(x,y,z) = x^2y\,\mathbf{i} - 8z\,\mathbf{j} + x^3yz\,\mathbf{k}$, compute div $\mathbf{F}$.

$$F(x,y,z) = \langle x^2y, -8z, x^3yz \rangle$$

$$\text{div } \vec{F} = \nabla \cdot \vec{F}$$

$$\nabla \cdot \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle x^2y, -8z, x^3yz \rangle$$

$$= 2xy + 0 + x^3y$$

$$= \boxed{2xy + x^3y}$$

Nice