Exam 2   Real Analysis 1   11/25/2002

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the limit L of a real-valued function f as x approaches ∞.

2. State the Extreme Value Theorem.

3. State the Mean Value Theorem.

4. Give an example of a function $f:\mathbb{R}\to\mathbb{R}$ which is discontinuous, but such that $|f(x)|$ is continuous.

5. Prove or give a counterexample: If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function for which f' is bounded, then f is bounded.

6. Give an example of a function $f:(0,1)\to\mathbb{R}$ for which f(x) is non-zero for infinitely many points in (0,1), but $\lim_{x\to a} f(x) = 0$ for all $a \in (0,1)$.

7. Prove or give a counterexample: If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function, $f(-1) = 3$ and $f(1)=3$, there must be a point in the interval $(-1,1)$ where f' is zero.

8. Prove that $f(x) = 1/x^3$ is continuous at $x=2$.

9. Prove that if $\lim_{x \to a} f(x)$ = A and $\lim_{x \to a} g(x)$ = B, then $\lim_{x \to a} (f - g)(x)$ = A-B.

10. Prove or give a counterexample: if $f:\mathbb{R}\to\mathbb{R}$ is an odd differentiable function, then f' is even.

Extra Credit (up to 5 points possible): Prove that $(\ln x)' = 1/x$.