Exam 1 Algebra & Trig 9/19/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Don't Panic.

1. Simplify the expression $\left(x^{-6}yz^{2}\right)^{-2}$ and write it without any negative exponents.

$(x^{-6}yz^{2})^{-2}$

$x^{12}y^{-2}z^{-4} = \dfrac{x^{12}}{y^{2}z^{4}}$

Good

2. Solve the system of equations:

$3a - b = 11$
$-a + 2b = -12$

$3a - 11 = b$
$3(2) - 11 = b$
$6 - 11 = b$
$-5 = b$

$-a + 2(3a - 11) = -12$
$-a + 6a - 22 = -12$
$5a = 10$
$a = 2$

I used substitution to solve this system of equations and I chose to start with the first equation and get b alone so I could plug it in for b in the second equation $(-a + 2(3a-11) = -12)$ then I simplified it and got a to be 2 and then I plugged that in as a in the first equation to get b which is -5 and a=2 & -5=b are my answers

Well done.

3. Solve the equation $\dfrac{x}{4} = \dfrac{x+3}{6} - 1$.

$$12 \cdot \frac{x}{4} = 12\left(\frac{x+3}{6} - 1\right)$$

$$3x = 2x + 6 - 12$$

$$x = -6$$

A

$$-\frac{6^{3}}{4_{2}} = \frac{-3}{6} - 1$$

$$-\frac{3}{2} = -\frac{1}{2} - \frac{2}{2}$$

ok

Good

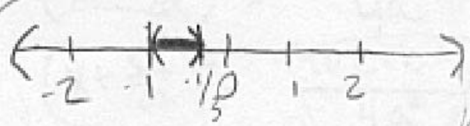
4. Solve the inequality $|3x + 2| < 1$, graph the solution on a number line, and write the solution in interval notation.

$$-1 < |3x+2| < 1$$

$$-1 < 3x + 2 < 1$$

$$-3 < 3x < -1$$

$$-1 < x < -\tfrac{1}{3}$$



$$(-1, -\tfrac{1}{3})$$

Great

I first wrote the equation out as $-1 < |3x+2| < 1$ and then you need to get rid of the absolute value signs which then gives you $-1 < 3x+2 < 1$ then you subtract two from each side which gives you $-3 < 3x < -1$ and then you divide by 3 and gives you your final equation of $-1 < x < -\frac{1}{3}$ then to graph it you put a paranthesis by one and $-\frac{1}{3}$ because it is not equal to! then you write it as $(-1, -\frac{1}{3})$

5. Divide $\frac{5+i}{3-2i}$ and write the answer in standard form.

Multiply top and bottom by the conjugate of the denominator:

$$\frac{5+i}{(3-2i)}\cdot\frac{(3+2i)}{(3+2i)} = \frac{15+10i+3i+2i^2}{9-4i^2} = \frac{13+13i}{13} = 1+i$$

$i^2 = -1$

6. Show that a quadratic equation of the form $ax^2+bx+c=0$ has solutions

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

$$a\left(x^2+\frac{b}{a}x+c\right)=0$$

$$a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right) = -c+\frac{b^2}{4a}$$

multiply by $1/a$

$$a\left(x+\frac{b}{2a}\right)^2 = \frac{-4ac+b^2}{4a}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = x = -\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

Well done!

Same as

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

7. Biff is an Algebra & Trig student at a large state university, and he's having some trouble. Biff says "So there was this problem on the homework, and it said we should say if 3 was a solution to the equation $x^2 - 2^x = 1$. But I totally don't know how to solve that equation, and I tried to for like three hours, because the teacher said that problem was about something really important and it'd probably be on the test. So now the test is Monday and I'm gonna have to waste my whole weekend trying to figure out how to solve it, because when I went to his office hours he wasn't even there."

Explain clearly to Biff how he should go about answering the question.

Listen Biff, instead of trying to solve the equation to see ~~if~~ 3 is an answer, you can look @ it the other way and see if 3 substitutes back into the equation as an answer

So does

$3^2 - 2^3 = 1$?

We can find out

3^2 is same as $(3)(3) = 9$

2^3 is same as $(2)(2)(2) = 8$

So does $9 - 8 = 1$

Yes Biff it does. So clearly it works as a solution

Exactly!

8. Find all solutions to the equation $3y^{2/3} + 2y^{1/3} - 8 = 0$.

let $x = y^{1/3}$: $3x^2 + 2x - 8 = 0$

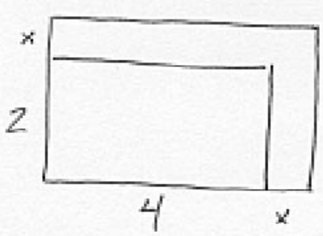
$$(3x-4)(x+2) = 0$$

$$x = 4/3 \quad \text{or} \quad x = -2$$

so $y^{1/3} = 4/3$ or $y^{1/3} = -2$

$$y = \frac{64}{27} \quad \text{or} \quad y = -8$$

9. If the length and width of a 4- by 2-inch rectangle are each increased by the same amount, the area of the new rectangle will be twice that of the original. What are the dimensions of the new rectangle (to two decimal places)?



$$\underbrace{(4+x)}_{\text{new width}}\underbrace{(2+x)}_{\text{new length}} = \underbrace{2}_{\text{twice}} \cdot \underbrace{4 \cdot 2}_{\text{old area}}$$

$$8 + 4x + 2x + x^2 = 16$$

$$x^2 + 6x - 8 = 0$$

Can't factor, so use quadratic formula:

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{68}}{2} \approx 1.12$$

(using the plus one - the other's negative and doesn't make sense here.)

So the dimensions of the new rectangle are $\approx 2 + 1.12 = 3.12$ inches by $\approx 4 + 1.12 = 5.12$ inches.

10. a) Write an absolute value equation that has $x = 2$ and $x = -2$ as solutions.

b) Write an absolute value equation that has $x = a$ and $x = b$ as solutions.

a) $|x| = 2$

I want all the stuff two units away from zero, so $|x - 0| = 2$ or $|x| = 2$.

b) $\left|x - \frac{a+b}{2}\right| = \frac{b-a}{2}$

I want stuff off to the right or left of a place half way between a and b, which is $\frac{a+b}{2}$. How far to the right or left is given by $b - \frac{a+b}{2}$ (think about the picture) or $\frac{2b}{2} - \frac{a+b}{2} = \frac{b-a}{2}$.

Check: $\left|a - \frac{a+b}{2}\right| \stackrel{?}{=} \frac{b-a}{2}$

$\left|\frac{2a}{2} - \frac{a+b}{2}\right| \stackrel{?}{=} \frac{b-a}{2}$

$\left|\frac{a-b}{2}\right| \stackrel{?}{=} \frac{b-a}{2}$

Because a-b is negative, its opposite is positive or.

$-\frac{(a-b)}{2} \stackrel{?}{=} \frac{b-a}{2}$

$\frac{b-a}{2} = \frac{b-a}{2}$ ✓

Extra Credit (5 points possible):

For what values of x and y will the inequalities

$$2x + y \geq 2$$
$$x - 3y \leq 1$$

both be satisfied? [Hint: Start by solving the system as if there were equals signs rather than inequalities, then see what you can do from there.]

$y = -2x + 2$

$y = \frac{1}{3}x - \frac{1}{3}$