Exam 2 Algebra & Trig 10/10/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. One bonus point for underlining this sentence to show you read the directions.

1. Write an equation for the line passing through the point (3,-1) with a slope of ½.

$y - y_1 = m(x - x_1)$

$y - (-1) = \frac{1}{2}(x - 3)$

$y + 1 = \frac{1}{2}(x - 3)$

Great

2. Find the center and radius of the circle with equation $x^2 + y^2 - 10y + 16 = 0$.

$x^2 + y^2 - 10y + 16 = 0$

$x^2 + (y^2 - 10y + 25) = -16 + 25$

$x^2 + (y - 5)^2 = 9$

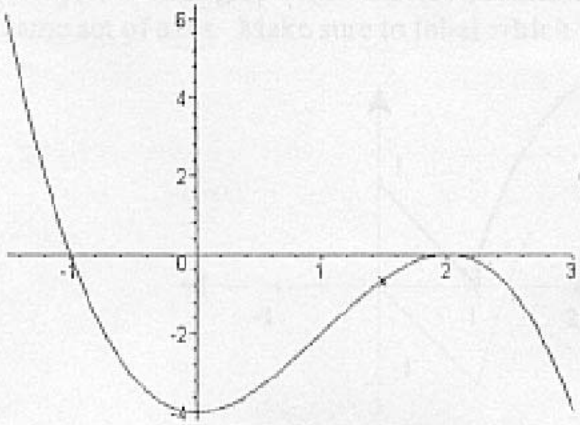
$x^2 + (y - 5)^2 = 3^2$

then the center is $(0, 5)$

the radius is 3

Excellent

3. Write a possible formula for the polynomial pictured below:



Negative so it goes up at the left / down at the right.

$y = -(x+1)(x-2)^2$

Makes it cross axis when $x = -1$

Makes it zero when $x = 2$

Squared makes it just touch the axis there.

4. If $f(x) = 3x$ and $g(x) = \sqrt{x+1}$, find:

a) $f(4)$

b) $g \circ f(2)$

c) $f(a+h)$

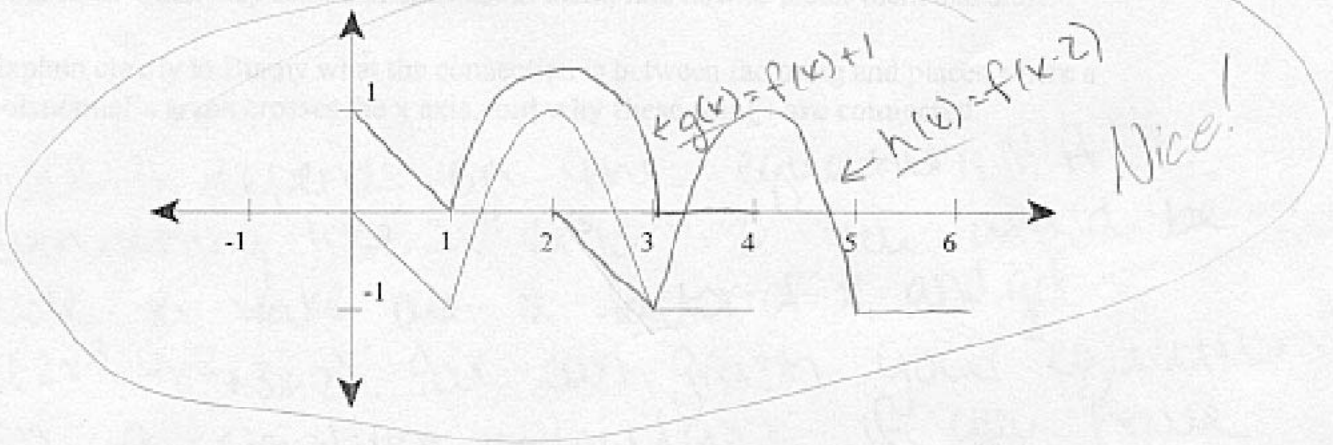
$f(4) = 3(4) = 12$

$g \circ f(a) = g(f(a)) = \sqrt{(3a)+1} = \sqrt{7}$

$f(a+h) = 3(a+h) = 3a+3h$

Great

5. If $f(x)$ has the graph shown below, sketch the graphs of $g(x) = f(x) + 1$ and $h(x) = f(x - 2)$ on the same set of axes. Make sure to **label which is which.**



6. If $f(x) = \dfrac{2x}{x+1}$, find $f^{-1}(x)$.

Solve for x:

$y = \dfrac{2x}{x+1}$

$y(x+1) = 2x$

$xy + y = 2x$

$xy - 2x = -y$

$x(y-2) = -y$

$x = \dfrac{-y}{y-2}$

Switch:

$f^{-1}(x) = \dfrac{-x}{x-2}$

Check:

$f(1) = \dfrac{2(1)}{(1)+1} = 1$ $\quad f^{-1}(1) = \dfrac{-(1)}{(1)-2} = 1$

$f(2) = \dfrac{2(2)}{(2)+1} = \dfrac{4}{3}$ $\quad f^{-1}\left(\dfrac{4}{3}\right) = \dfrac{-\frac{4}{3}}{\frac{4}{3}-2} = \dfrac{-\frac{4}{3}}{-\frac{2}{3}} = 2$ ✓

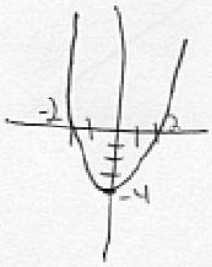
7. Bunny is a Precalc student at a large state university, and she's having some trouble. Bunny says "I can, like, always figure out where the polynomy things cross the x axis, but I never know what to do when they ask other stuff about them, like how to factor them and stuff."

Explain clearly to Bunny what the connection is between factoring and places where a polynomial's graph crosses the x axis, **and why these things are connected**.

The connection between factoring and where the polynomial crosses the x-axis is wherever the cross is, that's where it will equal zero.

Example:

$(x^2 - 4)$ if you graph (x^2-4) the graph looks something like this



It crosses the x axis at 2 and -2.

If you plug 2 and -2 into the equation, the answer you should get is zero.

$-2^2 = 4$ $\quad 2^2 = 4$

$4 - 4 = 0$ $\quad 4 - 4 = 0$

Excellent.

8. Find all solutions, real and complex, to the equation $x^4 + 2x^3 + x^2 + 8x - 12 = 0$.

Graph crosses at -3 and 1, so $(x+3)$ and $(x-1)$ are factors

$(x+3)(x-1) = x^2 + 2x - 3$

$$\begin{array}{r} x^2 \quad +4 \\ x^2+2x-3 \;\overline{)\; x^4+2x^3+x^2+8x-12} \\ -x^4-2x^3+3x^2 \\ \hline 4x^2+8x-12 \\ -4x^2-8x+12 \\ \hline 0 \end{array}$$

So $x^4+2x^3+x^2+8x-12 = (x^2+2x-3)(x^2+4)$

$= (x+3)(x-1)(x^2+4)$

$= (x+3)(x-1)(x+2i)(x-2i)$

So the solutions to the equation are

$x = -3$, $x = 1$, $x = -2i$, and $x = 2i$

9. If $f(x) = \dfrac{x(x-a)}{(x-b)(x-c)}$ for some distinct real constants a, b, and c, where does $f(x)$ have:

a) vertical asymptotes? $x = b$ and $x = c$

b) x-intercepts? $x = 0$ and $x = a$

c) horizontal asymptotes? $y = 1$

Values of x that make the denominator zero.

Values of x that make the numerator zero.

Since the numerator and denominator are both second degree, the horizontal asymptote is at the ratio of their coefficients (one over one in this case).

10. Decompose $\dfrac{5x^2+7x+6}{(x-1)(x+2)^2}$ into partial fractions.

Break into Parts:

$$\frac{5x^2+7x+6}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

Clear Denominators:

$$5x^2+7x+6 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

If $x=1$:

$$5(1)^2+7(1)+6 = A\cdot 9 + B\cdot 0 + C\cdot 0$$

so $9A=18$ and $A=2$

If $x=-2$:

$$5\cdot 4 + 7\cdot -2 + 6 = A\cdot 0 + B\cdot 0 + C\cdot -3$$

so $-3C=12$

and $C=-4$

If $x=0$:

$$0+0+6 = 4A - 2B - C$$

$$6 = 8 - 2B + 4$$

$$-6 = -2B \quad \text{and} \quad B=3$$

so $\dfrac{2}{x-1} + \dfrac{3}{x+2} + \dfrac{-4}{(x+2)^2}$