Exam 4 Algebra & Trig 12/5/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. (a) If vector $\mathbf{v}_1 = 5\mathbf{i} - 2\mathbf{j}$ and $\mathbf{v}_2 = -3\mathbf{i} + 2\mathbf{j}$, find $\mathbf{v}_1 - \mathbf{v}_2$.

(b) Find a unit vector $\mathbf{u}$ in the direction of the vector $\mathbf{v}_3 = -6\mathbf{i} + 8\mathbf{j}$.

$v_1 - v_2$

$(5i-2j)-(-3i+2j)$

$5i+3i=8i \qquad -2j-2j=-4j$

$\boxed{(a)\ 8i-4j}$

$(-6,8)$

$a^2+b^2=c^2$

$-6^2+8^2=c^2$

$36+64=c^2$

$100=c^2$

$c=10$

$\frac{1}{10}\cdot\frac{(-6,8)}{1}=$

$\frac{-6}{10},\frac{8}{10}$

$\boxed{(b)\ -\frac{3}{5}i+\frac{4}{5}j}$

Good

FOR (b) I simplified this

2. Find exact values for all real solutions to the equation $2\cos x - 1 = 0$.

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \cos^{-1}(\frac{1}{2})$

$x = \frac{\pi}{3} + 2\pi n$ for all integers n

$x = \frac{5\pi}{3} + 2\pi n$ for all integers n

$\frac{2\pi}{1} - \frac{\pi}{3}$

$\frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

Excellent

3. Convert the point with polar coordinates $(4, \pi/3)$ to rectangular coordinates.

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$$x = 4 \cdot \cos \tfrac{\pi}{3} = 4 \cdot \tfrac{1}{2} = 2$$

$$y = 4 \cdot \sin \tfrac{\pi}{3} = 4 \cdot \tfrac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$(2, 2\sqrt{3})$$

4. Verify the trig identity $(\sec t + 1)(\sec t - 1) = \tan^2 t$.

$$\tan^2 t = \frac{\sin^2 t}{\cos^2 t}$$

$$(\sec t + 1)(\sec t - 1)$$

$$= \sec^2 t - 1^2$$

$$= \frac{1}{\cos^2 t} - 1$$

$$= \frac{1}{\cos^2 t} - \frac{\cos^2 t}{\cos^2 t}$$

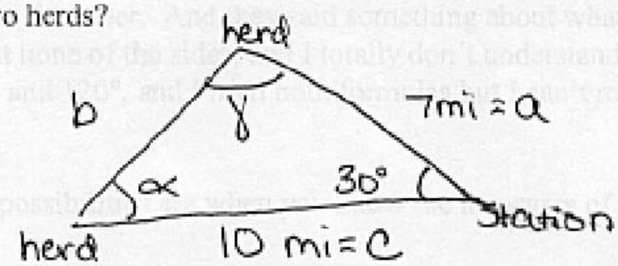
$$= \frac{1 - \cos^2 t}{\cos^2 t}$$

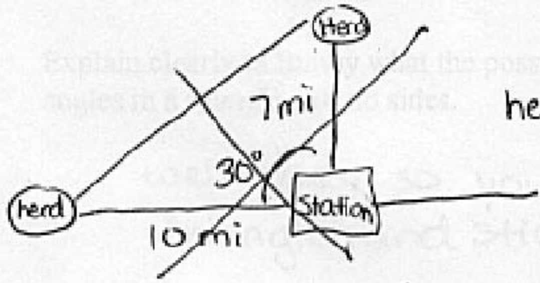
$$= \frac{\sin^2 t}{\cos^2 t}$$

$$= \tan^2 t$$

Great

5. A research station is tracking the elk population in a remote area. If one herd of elk is located 7 miles due north of the station, and another herd is located 10 miles from the station in a direction 30° west of due north (so that the lines between the two herds and the station form a 30° angle), how far apart are the two herds?





$b^2 = a^2 + c^2 - 2ac\cos 30°$ Well done

$b^2 = (7)^2 + (10)^2 - 2(7)(10)\cos 30°$

$b^2 = 49 + 100 - 140\cos 30°$

$b^2 = 149 - 121.24$

$\sqrt{b^2} = \sqrt{27.76}$

$\sqrt{4}\sqrt{7}$

$\boxed{\text{about } \sqrt{28} \text{ miles or } 2\sqrt{7} \text{ miles}}$

which would come out to be

5.3 mi on the calculator.

Yes!

6. Dan is calculus student is working out a problem and can't understand a line in the book that says $\sqrt{\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta} = \rho\sin\phi$. Explain to Dan how the expression on the left-hand side can be simplified to produce the expression on the right-hand side.

$\sqrt{\rho^2\sin^2\phi\,(\cos^2\theta) + \rho^2\sin^2\phi\,(\sin^2\theta)}$

use "$\cos^2\theta = 1-\sin^2\theta$"

$= \sqrt{\rho^2\sin^2\phi\,(1-\sin^2\theta) + \rho^2\sin^2\phi\,(\sin^2\theta)}$

multiplying inside

$= \sqrt{\rho^2\sin^2\phi - \cancel{\rho^2\sin^4\theta\phi} + \cancel{\rho^2\sin^4\phi\theta}}$

$= \sqrt{\rho^2\sin^2\phi}$ ...... $\sqrt{a^2} = a$, so...

$= \rho\sin\phi$

Good!

7. Bunny is having some trouble with solving triangles. She says "Ohmygod, this is so confusing. I mean, I get the two formulas, you know, the sin law and the cos law? But there was this problem in our study guide for our test, something about, like, different situations, and when to use one of the formulas instead of the other. And they said something about what to do when you know all three of the angles but none of the sides, and I totally don't understand. So, like, they said if the angles are 20°, 40°, and 120°, and I tried both formulas but I can't make them work. What do I do?"

Explain clearly to Bunny what the possibilities are when you know the measures of all three angles in a triangle, but no sides.

Bunny,

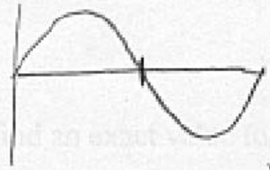
If all you have are the 3 angles then you need to use the law of sines. It would be impossible to use law of cosines w/out one side. Just remember the formula

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$. Actually Bunny now that I think about it more I think any time all you have are the angles you will not be able to figure out the sides. So just put that you cant decipher side lengths w/ only angle values. The only thing you would be able to tell is that the length opposite the largest angle is the largest side, and opposite the smallest angle is the smallest side.

Great

Excellent!

8. Find an exact value for sin 165°.



$\sin 165° = \sin(45° + 120°)$

$\sin(45°)\cos(120°) + \sin(120°)\cos(45°)$

$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}$

$\frac{-\sqrt{2}}{2} + \frac{\sqrt{6}}{2}$

$\frac{-\sqrt{2} + \sqrt{6}}{2}$

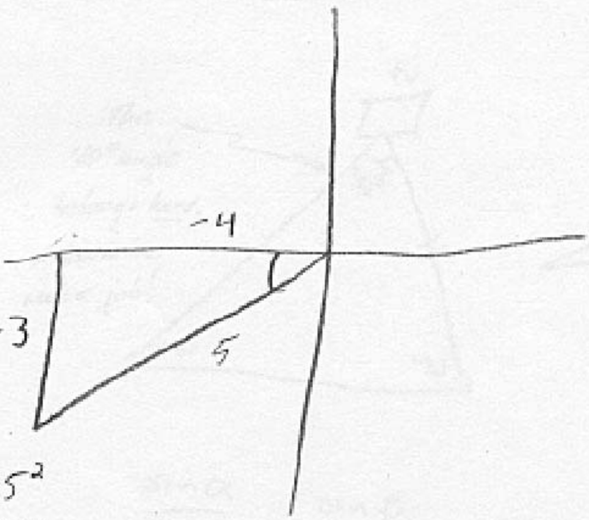
$\sin\theta$	
0	0
30	$\frac{1}{2}$
45	$\frac{\sqrt{2}}{2}$
60	$\frac{\sqrt{3}}{2}$
90	1
120	$\frac{\sqrt{3}}{2}$
135	$\frac{\sqrt{2}}{2}$
150	$\frac{1}{2}$
180	0

$\cos\theta$	
0	1
30	$\frac{\sqrt{3}}{2}$
45	$\frac{\sqrt{2}}{2}$
60	$\frac{\sqrt{1}}{2}$
90	0
120	$-\frac{1}{2}$

Great Job!

9. If θ is a third-quadrant angle for which $\cos\theta = -\frac{4}{5}$, find an exact value for $\sin 2\theta$.

sin cos tan



$-4^2 + b^2 = 5^2$

$16 + b^2 = 25$

$b^2 = 9$

$b = \pm\sqrt{9}$

$b = -3$

$\sin 2\theta = 2\sin(\theta)\cos(\theta)$

$= \frac{2}{1} \cdot \left(\frac{-3}{5}\right) \cdot \left(\frac{-4}{5}\right)$

$\frac{24}{25}$

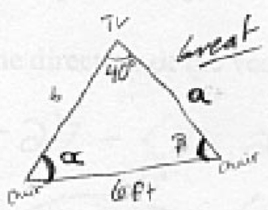
Well done

$\sin 2\theta = \frac{24}{25}$

10. Darcy is rearranging her living room. She wants two chairs to be exactly 6 feet apart, and both of the chairs to be the same distance from the television. Her husband Jon won't be happy if the angle between the lines joining each chair to the television is more than 40°. How far should the chairs be from the television?

$\alpha = 70°$

$\beta = 70°$



$\sin a = \frac{\sin \alpha}{a} \quad \frac{\sin \gamma}{c}$

$\frac{\sin \alpha}{a} = \frac{\sin \gamma}{c}$

$a = \frac{c \sin \alpha}{\sin \gamma}$

$a = \frac{6 \sin 70°}{\sin 40°}$

Chairs should be $\approx$ 8.8 ft. away from the television

Yes!