Practice Exam 4 Algebra & Trig 12/2/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

In most cases I'm just giving the answers here – as usual this alone, without justification, would get at most half credit.

1. Verify the trig identity $(\sin\theta + \cos\theta)^2 = 2\sin\theta\cos\theta + 1$

I'd mess with the left-hand side:

$$(\sin\theta + \cos\theta)^2 =$$
$$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta) =$$
$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta =$$
$$2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta =$$
$$2\sin\theta\cos\theta + 1 = 2\sin\theta\cos\theta + 1 \quad ✔$$

2. (a) If vector $\mathbf{v}_1 = 6\mathbf{i} - 2\mathbf{j}$ and $\mathbf{v}_2 = -\mathbf{i} + 3\mathbf{j}$, find $\mathbf{v}_1 + \mathbf{v}_2$.

(b) Find a unit vector $\mathbf{u}$ in the direction of the vector $\mathbf{v}_3 = -5\mathbf{i} + 12\mathbf{j}$.

(a) $5\mathbf{i} + 1\mathbf{j}$

(b) $\frac{-5}{13}\mathbf{i} + \frac{12}{13}\mathbf{j}$

3. Convert the point (-6,8) to polar coordinates.

$\left(10, \arctan\left(\frac{-4}{3}\right)\right)$

4. Find an exact value for cos 75°.

$\frac{\sqrt{6}-\sqrt{2}}{4}$

5. Jenny is trying to make a triangular sandbox for her kids because she has three boards. If one of the boards is 6 feet long, another is 7 feet long, and the last one is 8 feet long, what are the measures of the three angles in that triangle, rounded to the nearest degree?

47°, 58°, and 76° respectively.

6. If θ is a second-quadrant angle for which $\sin \theta = 3/5$, find an exact value for $\sin 2\theta$.

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7. Bunny is having some trouble with trig functions. She says "Okay, so like, I get how to do the thing where they say to verify a trig identity pretty good usually. But there was one on our practice test that said we were supposed to say whether this one was an identity *or not*, and I tried to make one side be like the other but I couldn't. So how do you *know* that there's not a way to do it? Do you just, like try for a few minutes and if you can't then you say it's wrong?"

Explain clearly to Bunny how you could determine when something is *not* a trig identity.

There are always lots of equally good ways to address one of these essay questions, but in this case I might say something like:

"Okay, Bunny, here's the key: If something is a trig identity, then it has to be true for every single value of the variable, right? So whether you put pi over three in, or negative pi over four, or zero, or any other number you put in, the left-hand side will be the same as the right-hand side. So if your first try to show it's an identity doesn't work out, you could pick your favorite value for the variable and plug it in. If the left-hand side and right-hand side don't match, then it wasn't an identity. If they do match, you can either pick another value to try in hopes of getting something that doesn't match, or you can go back and try again to show it's an identity. Eventually one or the other should work out.

Just remember that working for one number isn't enough to tell you it always works for every number, but failing for one number *does* tell you it doesn't work for every single number – so it wouldn't be an identity."

8. Find all real solutions to the equation $\sin x \cos x + \cos x = 0$.

$0 + n\pi$ for any integer n,
$\pi/2 + n\pi$ for any integer n,
$\pi + n\pi$ for any integer n,
and $3\pi/2 + n\pi$ for any integer n.
(There are other ways to write it, of course, but all of these possibilities have to be covered.)

9. Verify the trig identity $\dfrac{\tan x - \cot x}{\tan x + \cot x} = 1 - 2\cos^2 x$.

I'd work with the left, first writing everything in terms of sin and cos, then getting a common denominator, changing the division to multiplying by a reciprocal, and simplifying:

$$\frac{\tan x - \cot x}{\tan x + \cot x} =$$

$$\frac{\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}} =$$

$$\frac{\dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x}}{\dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}} =$$

$$\frac{\sin^2 x - \cos^2 x}{\cancel{\sin x \cos x}} \cdot \frac{\cancel{\cos x \sin x}}{\sin^2 x + \cos^2 x} =$$

$$\frac{\sin^2 x - \cos^2 x}{1} =$$

$$\left(1 - \cos^2 x\right) - \cos^2 x =$$

$$1 - 2\cos^2 x = 1 - 2\cos^2 x$$

10. Rescue workers are trying to locate some hikers who've gotten lost in a forest. From a brief cell-phone call they've determined that the hikers are due East from the park entrance, and 9 miles from the ranger station which is located 11 miles Northeast from the park entrance. How far are the campers from the entrance? [Hint: There are *two* possibilities, and you need to find both for full credit.]

Either 12.3 miles or 3.3 miles – this has to do with the ambiguous case using the Law of Sines.

Extra Credit (5 points possible):

If a triangle has sides of lengths 2, 3, and 4, what is the area of that triangle?