## Exam 1 Calculus $1 \quad$ 9/26/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Past results cannot guarantee future returns.

1. State the formal definition of the derivative of a function $f(x)$.
2. Given the graph of $g(x)$ shown below:

a) State the values of $x$ for which $g(x)$ is not continuous.
b) State the values of $x$ for which $g(x)$ is not differentiable.
3. Given the following table of information about $f(x)$, write an equation for the line tangent to $f(x)$ when $x=2$.

| $x$ | $f(x)$ | $f^{\prime}(x)$ | $f^{\prime \prime}(x)$ |
| :---: | :---: | :---: | :---: |
| 0 | 3 | 2 | 1 |
| 1 | 5 | 3 | -2 |
| 2 | 8 | -1 | -1 |
| 3 | 6 | -2 | 0 |

4. Given the graph of $g(x)$ below, sketch the graphs of $g^{\prime}(x)$ and $g^{\prime \prime}(x)$ on the same set of axes.

5. Use the definition of the derivative to find $f^{\prime}(2)$ for the function $f(x)=x^{2}-3 x$.
6. A graph of $f(x)=x^{x}$ is shown below. Estimate the value of $f^{\prime}(1)$ accurate to at least the nearest hundredth.

7. In USA Today on July $24^{\text {th }}, 2003$, a story about America Online included the following:
"Another worry: a 9\% year-over-year growth slowdown in the cable unit's high-speed Internet subscriptions, up 170,000 in the quarter to 2.9 million. That business is under attack as phone companies lower DSL prices"

If $h(x)$ is a function giving the number of America Online's high-speed internet subscriptions, what is the quote telling you about the signs of $h(x), h^{\prime}(x)$, and $h^{\prime \prime}(x)$ for the last quarter?
8. Biff is a business calculus student at a large state university, and he's having some trouble. Biff says "So our professor makes less than no sense, I think he hates that he has to teach us, and the grad student who teaches the discussion section is a total freak. He was, like, ranting about something yesterday, and it was like about how if there's a company, and its profits are negative, but the second derivative of its profits are positive, then... Well, I didn't totally get it. He was either saying they'd eventually make a profit, or they didn't, and I don't know which. But he said if we could write, like, a clear explanation of why it happened that way or whatever, then he'd give us extra credit. I think I'm gonna need it."

Explain clearly to Biff how he should go about answering the question - that is, what can (or can't) be deduced from the knowledge that $p(t)$ is negative and $p^{\prime \prime}(t)$ is positive.
9. If $f(x)$ and $g(x)$ are two functions and $f(2)=g(2)$, then will $f^{\prime}(2)=g^{\prime}(2)$ ? Either say why it must be so, or give an example of functions for which it is not.
10. Suppose that a section of railroad track is shaped like the line $y=-x$ for values of $x$ less than 1. You want to join to this a curved section of track shaped like a parabola which passes through both the points $(0,0)$ and $(1,1)$ and so that the resulting track is differentiable (that is, so that the slope of the line matches the slope of the parabola where they meet). Find an equation for the parabola which does this.
[Hint: Notice that if $p(x)=a x^{2}+b x+c$, then $p^{\prime}(x)=2 a x+b$.]

Extra Credit (5 points possible):
If $f(x)$ is an even function, what can you say about $f^{\prime}(x)$ ? How do you know this?

