Exam 1 **Calculus 1** **9/26/2003**

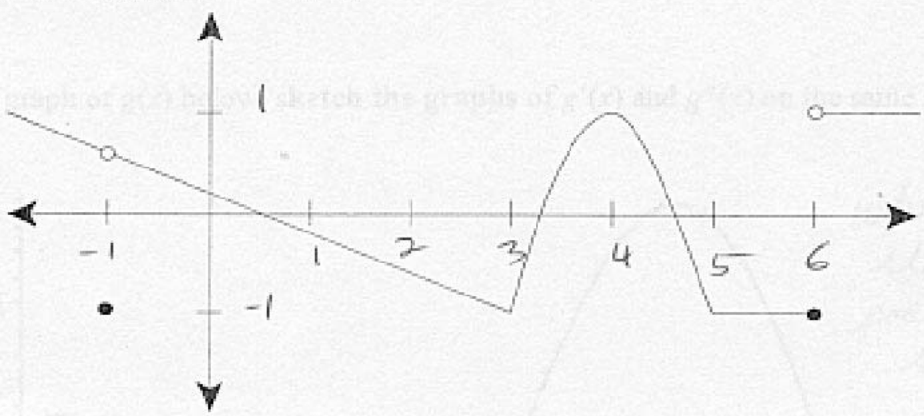
Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Past results cannot guarantee future returns.

1. **State the formal definition** of the derivative of a function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Yes

2. Given the graph of $g(x)$ shown below:



a) State the values of x for which $g(x)$ is not continuous.

$\boxed{x = -1, \quad x = 6}$: point does not agree with the limit as x approaches -1 and limits don't agree at x=6

b) State the values of x for which $g(x)$ is not differentiable.

$\boxed{x = -1, \; x = 3, \; x = 5, \; x = 6}$

- can't be differentiable if not continuous
- not differentiable at corners

Great

3. Given the following table of information about $f(x)$, **write an equation** for the line tangent to $f(x)$ when $x = 2$.

x	$f(x)$	$f'(x)$	$f''(x)$
0	3	2	1
1	5	3	-2
2	8	-1	-1
3	6	-2	0

$y = mx + b$

$m = -1$

$y = -x + b$

this line through $(2, 8)$

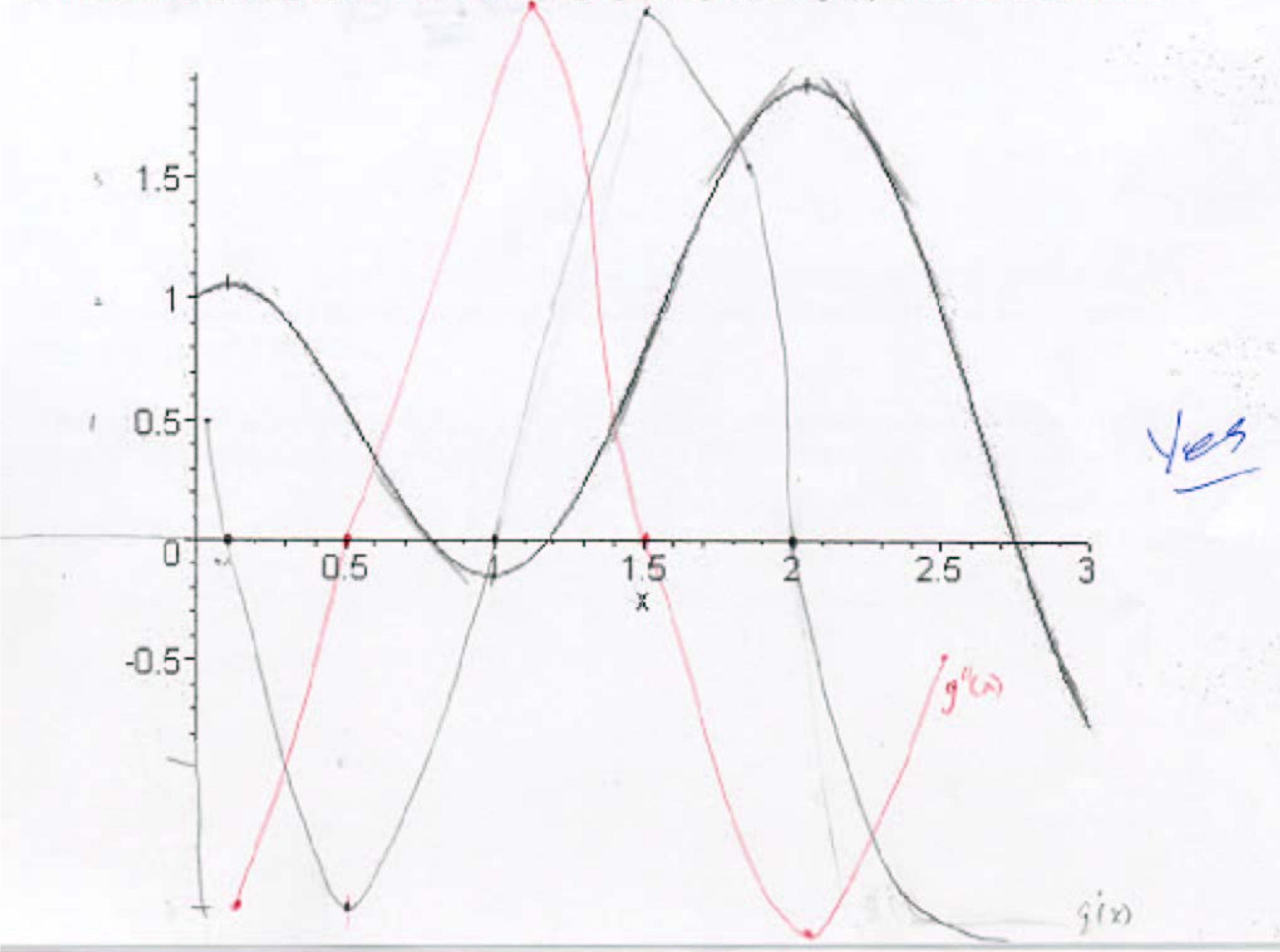
$8 = -2 + b$

Great

$b = 10$

$\boxed{y = -x + 10}$

4. Given the graph of $g(x)$ below, **sketch the graphs** of $g'(x)$ and $g''(x)$ on the same set of axes.



5. **Use the definition of the derivative to find $f'(2)$ for the function $f(x) = x^2 - 3x$.**

we have,

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^2 - 3(2+h) - ((2)^2 - 3(2))}{h}$$

$$= \lim_{h \to 0} \frac{4 + 4h + h^2 - 6 - 3h - (4-6)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + h - 2 - (-2)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + h - 2 + 2}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h+1)}{\cancel{h}}$$

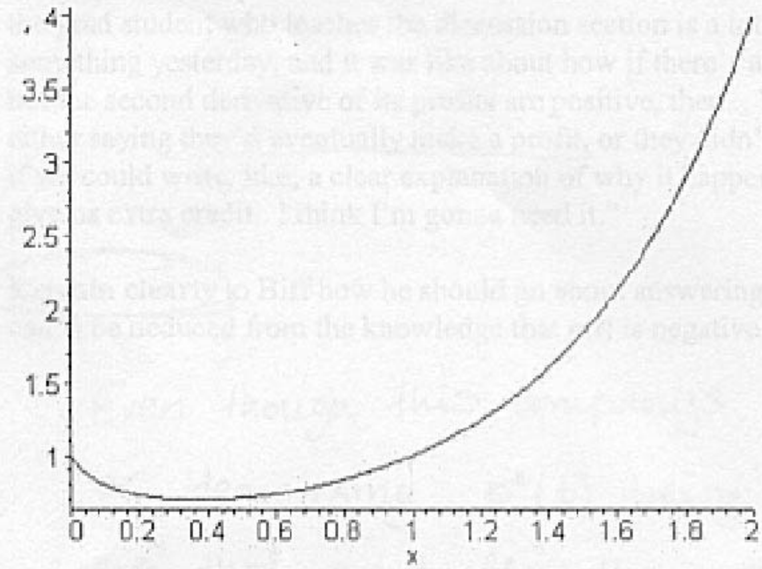
$$= \lim_{h \to 0} h + 1$$

$$= 0 + 1$$

$$\therefore \underline{f'(2) = 1}$$

Great Job

6. A graph of $f(x) = x^x$ is shown below. **Estimate** the value of $f'(1)$ accurate to at least the **nearest hundredth.**



x	$f(x)$
1.01	1.0101
1.001	1.001
1.0001	1.0001
1.00001	1.00001

$f'(x) = \dfrac{1.0101 - 1.001}{1.01 - 1.001} \approx 1.011$

$f'(x) = \dfrac{1.001 - 1.0001}{1.001 - 1.0001} = 1.00$

$f'(x) \approx 1.00$

Excellent

$\boxed{f'(1) \approx 1.00}$

7. In *USA Today* on July 24th, 2003, a story about America Online included the following:

"Another worry: a 9% year-over-year growth slowdown in the cable unit's high-speed Internet subscriptions, up 170,000 in the quarter to 2.9 million. That business is under attack as phone companies lower DSL prices"

If $h(x)$ is a function giving the number of America Online's high-speed internet subscriptions, **what is the quote telling you about the signs of** $h(x)$, $h'(x)$, and $h''(x)$ for the last quarter?

$h(x) = +$

$h'(x) = +$

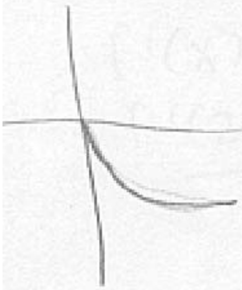
$h''(x) = -$

Nice Job

- 2.9 million is positive
- up 170,000 means it's increasing
- 9% slowdown means increasing @ a slower rate

8. Biff is a business calculus student at a large state university, and he's having some trouble. Biff says "So our professor makes less than no sense, I think he hates that he has to teach us, and the grad student who teaches the discussion section is a total freak. He was, like, ranting about something yesterday, and it was like about how if there's a company, and its profits are negative, but the second derivative of its profits are positive, then... Well, I didn't totally get it. He was either saying they'd eventually make a profit, or they didn't, and I don't know which. But he said if we could write, like, a clear explanation of why it happened that way or whatever, then he'd give us extra credit. I think I'm gonna need it."

Explain clearly to Biff how he should go about answering the question – that is, what can (or can't) be deduced from the knowledge that $p(t)$ is negative and $p''(t)$ is positive.



If $p(t)$ is negative, then the company is losing money. If $p''(t)$ is positive then profits are dropping more slowly than they used to be.

Yes.

9. If $f(x)$ and $g(x)$ are two functions and $f(2) = g(2)$, then will $f'(2) = g'(2)$? Either say why it must be so, or give an example of functions for which it is not.

$f(x) = x^2 + 2$ $\qquad g(x) = 2x^2 - 2$

$f(2) = 2^2 + 2$

$f(2) = 6$ $\qquad g(2) = 6$

So, if $f(x) = x^2 + 2$, $f(2)$ would $= 6$ and if $g(x) = 2x^2 - 2$ $g(2) = 6$ $\Rightarrow$ But the Derivative of $f(x)$ is $f'(x) = 2x$ and the derivative of $g(x)$ is $g'(x) = 4x$ $\Rightarrow$

So when you plug in 2 in the derivative fontions you would get

$f'(2) = 2x$

$f'(2) = 2 \times 2 = 4$

$g'(2) = 4x$

$g'(2) = 4(2) = 8$

$\boxed{4 \neq 8}$ NOT EQUAL

not always equal - it might work for some super confusing function but not in mine

Excellent

10. Suppose that a section of railroad track is shaped like the line $y = -x$ for values of x less than 1. You want to join to this a curved section of track shaped like a parabola which passes through both the points (0,0) and (1,1) and so that the resulting track is differentiable (that is, so that the slope of the line matches the slope of the parabola where they meet). Find an equation for the parabola which does this.
[Hint: Notice that if $p(x) = ax^2 + bx + c$, then $p'(x) = 2ax + b$.]

Let us assume that the equ^n of parabola is
$p(x) = ax^2 + bx + c$ $\quad \therefore p'(x) = 2ax + b$

We know,
$p'(x)$ = slope of parabola
or, slope of parabola = slope line which is -1

$\therefore 2ax + b = -1$ ----- (1)
or, $0 + b = -1$ $\quad \therefore \underline{b = -1}$

Also parabola passes through (0,0) and (1,1)

$\therefore p(0) = c$ $\qquad$ And, $p(1) = a + b + c$ $\qquad \because c = 0$ and $p(1) = 1$
$\therefore 0 = c$ $\qquad \therefore a + b = 1$ --- (1)
$\therefore a - 1 = b$
$a = 2$

$\therefore$ Equation of parabola is

$$\boxed{p(x) = 2x^2 - x}$$

Well done