## Exam 2 Calculus 1 11/7/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. I promise it's revenue-neutral.

1. If $\mathrm{f}(x)=x^{3}-6 x+1$, find the exact locations of the local maximum and minimum points of $\mathrm{f}(x)$.
2. Consider the curve shown below, with parametric equations $x(t)=2 t^{2}-t$ and $y(t)=3 t^{3}-t$. What are the exact coordinates of the point where a line tangent to the curve is vertical?

3. Prove the sum rule for derivatives, i.e. that $(\mathrm{f}+\mathrm{g})^{\prime}(x)=\mathrm{f}^{\prime}(x)+\mathrm{g}^{\prime}(x)$.
4. Find the slope of the line tangent to $x^{2} y+y^{2}=10$ at the point $(3,1)$.

5. If an anesthetic used to sedate large mammals during surgeries has a peak effectiveness of 100, which occurs 30 minutes after the drug is administered, find a curve of the form $\mathrm{f}(t)=a t e^{-b t}$ to represent the effectiveness of the sedative $t$ minutes after administration.
6. Show that the derivative of $\tan x$ is $\sec ^{2} x[$ remember $\tan x=\sin x / \cos x$ and $\sec x=1 / \cos x]$.
7. Prove the quotient rule for derivatives [you may use the product rule in doing so if you like].
8. Jim wants to build a rectangular parking lot on a piece of property shaped like the region between the curve $y=x^{3}$, the line $y=8$, and the positive $y$ axis (where the units are measured in increments of how far Jim can walk between puffs on his cigar). What dimensions should the parking lot have in order to maximize its area?
9. Biff is a business calculus student at a large state university, and he's having some trouble. Biff says "So I hate our professor, and like half the people in the class dropped already but I can't 'cause my dad will kill me. So we've got this test coming up, and the professor was saying something about how this function x plus $\sin \mathrm{x}$ was really important, and like we're supposed to find its max and min values or something, I don't know. I really couldn't figure out what he was saying about it at all. But I'm pretty sure it's gonna be on the test, so I better know how to do it. I know you always take the derivative and put it equal zero, but I don't know how to make that work on this one."

Explain clearly to Biff what the situation is regarding maximum and minimum values of the function $\mathrm{f}(x)=x+\sin x$.
10. If $\mathrm{f}(x)$ is a third degree polynomial with a point of inflection at the origin, what can you say about the formula and graph of $\mathrm{f}(x)$ ? Be clear about how you reach your conclusions.

Extra Credit (5 points possible):
If $\mathrm{f}(x), \mathrm{g}(x)$, and $\mathrm{h}(x)$ are differentiable functions, find a formula for the derivative of $\mathrm{f}(\mathrm{g}(\mathrm{h}(x)))$.

