Exam 2 Calculus 3 10/1/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. It's a slippery slope.

1. State the formal definition of the partial derivative of the function $f(x,y)$ with respect to y.

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{x^2 + y^2}$ does not exist.

3. If **v** = $<-5, 12>$ and $f(x,y) = 4x - xy^2$, what is the directional derivative of f in the direction of **v**?

4. If $w = f(x,y,z)$, $x = x(t)$, $y = y(t)$, and $z = z(t)$, state the appropriate version of the chain rule for $\frac{dw}{dt}$. **Make it clear which of your derivatives are partials.**

5. If $f(x,y) = x^3y - y^2$, in which direction is the directional derivative at the point (-1,2) greatest, and what is the value of that directional derivative?

6. Find the point on the plane $x + y + z = 1$ closest to the point $(3, 0, 0)$.

7. Bunny is a calc 3 student at a large state university and she's having some trouble. Bunny says "Ohmygod, I am so totally confused by this class. I mean, I can work out a lot of the problems, but I totally don't understand what any of it means. I guess it doesn't really matter, since our exams are all multiple choice, but it really seems like some day I might need to know why some of this stuff works. Like, I totally know that when the question says to find the direction of greatest increase, you figure out the gradient thing and that's the answer. But why? I have no clue, even if I'm getting an A."

Explain clearly to Bunny why the gradient is connected to a direction of greatest increase.

8. Find the maximum value(s) of the function $f(x,y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 4$.

9. For which values of b is $f(x,y) = x^2 + bxy + y^2$ a hyperbolic paraboloid?

10. Will an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=\mathbf{1}$ necessarily have a point (x_0,y_0,z_0) where its tangent plane has a normal vector parallel to <1,1,1>? How do you know?

Extra Credit (5 points possible):

Generalize your answer to problem 6 for a point of the form (*a*,0,0).