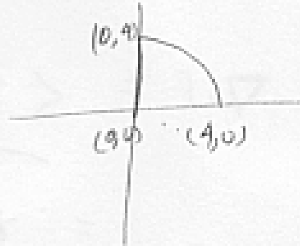
Quiz 4 Calculus 3 11/14/03

Each problem is worth 5 points. For full credit indicate clearly how you reached your answer.

1. Compute $\oint_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F}(x,y) = \langle xy, 2x^2\rangle$, where C is the path beginning with a line segment from (0,0) to (4,0), followed by an arc of a circle (centered at the origin) from (4,0) to (0,4), and finally the line segment from (0,4) to (0,0).



$$\oint_C F\cdot dr = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_D (4x - x)\, dA = \iint_D 3x\, dA$$

$$= \int_0^{\frac{\pi}{2}}\int_0^4 3\cdot r\cos\theta\cdot r\,dr\,d\theta \quad +$$

$$= \int_0^{\frac{\pi}{2}}\int_0^4 3r^2\cos\theta\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cancel{3}r^3}{\cancel{3}}\Big|_0^4 \cos\theta\,d\theta = 4^3\int_0^{\frac{\pi}{2}} \sin\theta\,d\theta$$

$$= 4^3 = \underline{64}$$

Excellent

2. Find div <3xyz, y^2, x^3y^4>.

$$\operatorname{div}\vec{F} = \langle \partial/\partial x, \partial/\partial y, \partial/\partial z \rangle \cdot \langle 3xyz, y^2, x^3y^4 \rangle$$

$$= 3yz + 2y + 0$$

$$\boxed{= 3yz + 2y}$$

Great

3. Find curl <2x, -z, y>.

P Q R

$$\text{curl} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & -z & y \end{vmatrix} = 1i + 0j + 0k - (0k - 1i + 0j)$$

$$= 2i$$

$$\text{curl}\langle P, Q, R \rangle = \langle 2, 0, 0 \rangle$$

$$\left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

$$= \langle 2, 0, 0 \rangle$$

Great