Quiz 5 **Calculus 3** **Due 11/19/03**

Each problem is worth 5 points. For full credit indicate clearly how you reached your answer.

1. Find the surface area of a sphere with radius 1 using the formula from section 16.6.

$$\text{Surface Area} = \iint_D |\vec{r}_u \times \vec{r}_v| \, dA$$

We can parametrize like spherical:

$$x(u,v) = 1 \cdot \cos u \sin v$$
$$y(u,v) = 1 \cdot \sin u \sin v$$
$$z(u,v) = 1 \cdot \cos v$$

so $\vec{r}(u,v) = \langle \cos u \sin v, \sin u \sin v, \cos v \rangle$

and $\vec{r}_u(u,v) = \langle -\sin u \sin v, \cos u \sin v, 0 \rangle$

$\vec{r}_v(u,v) = \langle \cos u \cos v, \sin u \cos v, -\sin v \rangle$

Now we cross 'em: $$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin u \sin v & \cos u \sin v & 0 \\ \cos u \cos v & \sin u \cos v & -\sin v \end{vmatrix}$$

$$= -\cos u \sin^2 v \, \vec{i} + 0 \, \vec{j} - \sin^2 u \sin v \cos v \, \vec{k}$$
$$- (0 \, \vec{i} + \sin u \sin^2 v \, \vec{j} + \cos^2 u \sin v \cos v \, \vec{k})$$

$$= \langle -\cos u \sin^2 v, -\sin u \sin^2 v, (\sin^2 u + \cos^2 u) \cdot -\sin v \cos v \rangle$$

And take the magnitude of this:

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{\cos^2 u \sin^4 v + \sin^2 u \sin^4 v + \sin^2 v \cos^2 v}$$

$$= \sqrt{(\sin^2 u + \cos^2 u) \sin^4 v + \sin^2 v \cos^2 v}$$

$$= \sqrt{\sin^2 v (\sin^2 v + \cos^2 v)}$$

Notice there are some (okay) domain issues.

$$= \sin v$$

Finally integrate: $$\int_0^{2\pi} \int_0^{\pi} \sin v \, dv \, du = \int_0^{2\pi} 2 \, du$$

$$= 4\pi$$

2. Compute $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where $F(x,y,z) = \langle 2x,-z,y\rangle$ and S is the portion of $x = 4 - y^2 - z^2$ within the cylinder $y^2 + z^2 = 4$.

I. Parametrize: $x(u,v) = 4-u^2-v^2$

$y(u,v) = u$

$z(u,v) = v$

so $\vec{r}(u,v) = \langle 4-u^2-v^2, u, v\rangle$

II. Build $\vec{F}(\vec{r}(u,v))$: $\vec{F}(\vec{r}(u,v)) = \langle 2(4-u^2-v^2), -v, u\rangle$

$= \langle 8-2u^2-2v^2, -v, u\rangle$

III. Build $\vec{r}_u'(u,v)\times\vec{r}_v(u,v)$: $\vec{r}_u(u,v) = \langle -2u, 1, 0\rangle$

$\vec{r}_v(u,v) = \langle -2v, 0, 1\rangle$

$$\vec{r}_u\times\vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2u & 1 & 0 \\ -2v & 0 & 1 \end{vmatrix}$$

$= \langle 1, 2u, 2v\rangle$

IV. Put 'em together: $\iint_D \langle 8-2u^2-2v^2, -v, u\rangle\cdot\langle 1, 2u, 2v\rangle dA$

$$= \iint_D (8-2u^2-2v^2-2uv+2uv)\,dA$$

$$= \int_0^{2\pi}\int_0^2 (8-2r^2)\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[4r^2-\tfrac{1}{2}r^4\right]_0^2 d\theta$$

$$= \int_0^{2\pi}(16-8)\,d\theta$$

$$= 8\theta\Big|_0^{2\pi}$$

$$= 16\pi$$