

Exam 1 Calc 2 9/16/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **State the formula** for volume by cylindrical shells when the region between $f(x)$ and the x -axis from $x = a$ to $x = b$ is rotated around the y -axis.

2. **Set up an integral** for the average value of the function $g(t) = te^{-t^2}$ on the interval $[0,5]$.

3. **Find** the area of the region between $y = 6 - x^2$ and $y = x$.

4. If $F(x) = \int_0^x \sin^2 t \, dt$, what is $F'(x)$?

5. A spring has a natural length of 16 inches, and 60 pounds hold it stretched to a length of 20 inches. How much work is done in stretching the spring from a length of 20 inches to a length of 22 inches?

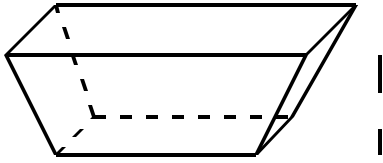
6. Evaluate $\int_0^{\pi/6} \sin \theta \cos^4 \theta d\theta$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is the most totally confusing experience in my life. The professor gave us this huge u -substitution assignment, and she really emphasized that it's important, so I want to make sure to get it all right, you know? But so it's like 200 integrals, and half of them are the even-numbered problems, right? And she said it was really important that we check our answers, but my book doesn't have answers for the even ones. And so my friend asked the professor in class about it, 'cause I was afraid to, and the professor got really excited and kept saying that of course we could check our answers even if they're even. Is she just crazy?"

Explain clearly to Bunny how she might be expected to check her answers on such problems, even when the answers to the evens aren't in the back of the book.

8. Jon has decided to build a rampart around his house shaped like the solid formed by rotating the region under $f(x) = 9 - x^2$ but above the x axis around the axis $x = 30$ (with the scale in feet). **Set up an integral** for the volume of this rampart.

9. Suppose a pit shaped like an upside-down frustum of a pyramid is filled with water. If the pit has a square with side length b at the top and a square with side length a at the bottom, and a depth of h (with all of these measurements in meters, and water with a density of 1000kg/m^3), **set up an integral** for the work required to pump all the water out over the top edge of the pit.



10. a) Let R be the first-quadrant region bounded by the y -axis, the curve $y = x^2$, and the line $y = 1$. Find the volume of the solid resulting when R is rotated about the line $x = -2$.

b) Find a positive value of b for which the volume obtained when rotating the region R from part a about the line $y = b$ is the same as that obtained in part a.

Extra Credit (5 points possible):

Suppose the region inside the triangle with vertices $(1,0)$, $(2,0)$, and $(1,1)$ is rotated around the line $y = x$. Describe the solid formed, and find its volume.