Exam 3 Calc 2 11/4/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formula for arc length in polar coordinates.

2. Determine whether $y = 2 + 5e^{2x+3}$ is a solution to the differential equation $\dfrac{dy}{dx} = 2y - 4$.

3. Find the exact coordinates of all points on the curve with parametric equations $x(t) = t^3 - 12t$, $y(t) = 10 - t^2$ where the tangent line is vertical.

4. Find a general solution to the differential equation $\frac{dH}{dt} = k(212 - H)$.

5. Classify the graph with equation $4x^2 + 9y^2 + 108 = 72y$ as a parabola, ellipse, or hyperbola, and sketch the graph, labeling the exact coordinates of at least four points.

6. A company is trying to market an insecticide to combat the invasion of Asian beetles in Iowa, claiming that it will kill the beetles at an increasing rate every year until they are eliminated. Suppose that the situation is modeled by the differential equation $\frac{dP}{dt} = 0.1P - 4t$. Use Euler's method with $\Delta t = 5$ to approximate the Asian beetle population 15 years from the present, provided that the present population is 800 (with all the population numbers actually measured in millions of beetles).

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is so confusing. Our professor was making this big deal in lecture yesterday about polar coordinates, but I have no idea what his point was. He kept saying something about uniqueness, but I really didn't get it. He seemed to think it was important, though, so probably I should figure it out."

Explain clearly to Bunny what is or is not unique about the coordinates of a point when expressed in polar coordinates.

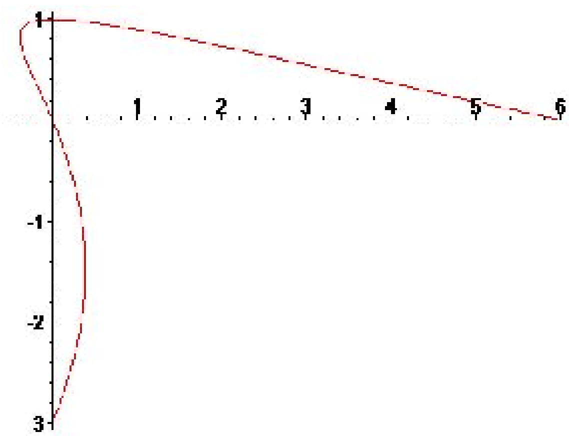
8. Find the area of the region inside the graph with polar equation $r = 2 + 4\cos\,\theta$ but outside the graph with polar equation $r = 4$.

9. Suppose that you know a function $y = f(x)$ is a solution to the differential equation

$$ay' + by = 0$$

where a and b are constants. Show that any other function of the form $y = A \cdot f(x)$, where A is a constant, will also be a solution to that differential equation.

10. Find the area of the region bounded by the graph of the parametric functions $x(t) = t^3 - t$, $y(t) = 2t - t^2$ and the x-axis.



Extra Credit (5 points possible):

Every morning Jon feeds his four cats half a can of cat food. The can of cat food is shaped like a cylinder 1 unit high and 2 units in diameter. He tries to divide it evenly among the cats in the following manner: First he cuts the cylinder in half, bisecting the circular face. Then he makes a second cut perpendicular to the first, as close to bisecting the line segment of the first cut as possible. Then he makes two more cuts, each at a 45° angle to the two previous cuts. He then puts four of the resulting wedges in dishes for the four cats, and the rest in the refrigerator. The problem is, sometimes his second cut is a little bit off from the exact middle of the first cut, which means the wedges produced are not all equal. If the second cut intersects the first cut at a distance k off from the center (but the third and fourth cuts still pass through that point of intersection, so that all of the angles formed are multiples of 45°), find the area of the smallest wedge (which is the one Nil gets).