

**Exam 4    Calc 2    12/2/2005**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the first 3 partial sums of the series  $\sum_{n=1}^{\infty} \frac{1}{3^n}$ .

2. Determine whether the sequence  $\left\{ \frac{3+5n^2}{n+n^2} \right\}$  converges or diverges, and if it converges find its limit.

3. Determine the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{3^n}$ .

4. Determine whether the series  $\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1}$  converges or diverges.

5. Determine whether the series  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$  converges or diverges.

6. Determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$  converges or diverges.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this is so crazy. Pretty much the only thing I've figured out so far about this series crap is that the series one over  $n$  is supposed to diverge, right? So we had this quiz and it had  $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$  on it, and I saw the one over  $n$  in there and knew it had to diverge, right? So I said it compared to the one over  $n$  thing so it had to diverge, but the T.A. wrote a bunch of crap I didn't understand, and I got no credit at all. What's up with that?"

**Explain clearly** to Biff what's wrong with his version of "comparison".

8. Determine the interval of convergence for the power series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ .

9. Use a 9<sup>th</sup> degree polynomial to approximate  $\int_0^1 \sqrt{1-x^3} dx$ . [Hint: Starting by finding a series for  $f(x) = \sqrt{1+x}$  might be a good plan].

10. If  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  are both convergent series, will  $\sum_{n=1}^{\infty} (a_n \cdot b_n)$  converge? Why or why not?

Extra Credit (5 points possible):

Determine whether the series  $1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} - \frac{1}{64} + \frac{1}{128} + \dots$  (so all the denominators are powers of 2, but the sign on every fourth term is negative) converges or diverges, and if it converges find the limit.