

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Determine whether the series  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$  converges or diverges.

$\checkmark$  A.S.T  
 decrease?  $\frac{1}{n \ln n} > \frac{1}{(n+1) \ln(n+1)}$  almost  $\ln(n+1) > n$   
 $\frac{(n+1) \ln(n+1)}{n \ln n} > \frac{n}{n \ln n}$   $1 > 0$   
 $\frac{(n+1) \ln(n+1)}{n \ln n} > \frac{1}{\ln(n+1) + \ln(n+1)}$   $\frac{1}{n+1} > n$   
 $n \ln(n+1) + \ln(n+1) > n \ln n$   $\ln(n+1) > \ln n$   
 $\ln(n+1) (n+1) > n \ln n$   $n+1 > n$   
 $\frac{\ln(n+1) (n+1)}{\ln n} > n$   $\frac{n+1}{n} > n$   
 $\frac{\ln(n+1) \cdot (n+1)}{\ln n} > n \ln n$   $\ln(n+1) \cdot (n+1) > n \ln n$  Great!  
 $\frac{1}{n \ln n} > \frac{1}{\ln(n+1) \cdot (n+1)}$   
 $\checkmark$   $\lim_{n \rightarrow \infty} \frac{1}{n \ln n} = 0$  so its  $\lim$  is 0  
 $\lim_{n \rightarrow \infty} \frac{1}{n \ln n} = 0$  because its decreasing and  $\lim_{n \rightarrow \infty} = 0$ .

2. Determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$  converges or diverges.

$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \begin{cases} > 1 \text{ or } \infty, \text{ diverges} \\ < 1, \text{ converges absolutely} \end{cases}$   
 $\lim_{n \rightarrow \infty} \left| \frac{\frac{1}{(2(n+1))!}}{\frac{1}{(2n)!}} \right| = \lim_{n \rightarrow \infty} \left| \frac{1}{(2n+2)!} \cdot \frac{(2n)!}{1} \right| = \lim_{n \rightarrow \infty} \frac{1}{(2n+2)(2n+1)(2n)!} \cdot \frac{(2n)!}{1}$   
 can get rid of "!" because all positives  
 $= \lim_{n \rightarrow \infty} \frac{1}{(2n+2)(2n+1)} = 0$   $0 < 1$  so, Nice!

Therefore, by the Rat. Test, it converges absolutely.