Exam 1 Calc 3 9/23/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to y.

2. Compute the directional derivative of the function g(x, y) = xy^3 - 2x at the point (2,3) in the direction of the vector $4\vec{i} - 3\vec{j}$.

3. If f is a function of the two variables x and y, and x and y are in turn both functions of the variable t, write the appropriate version of the chain rule for $\frac{df}{dt}$. Make clear which, if any, of your derivatives are partials.

4. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2+2xy+y^2}{x^2+y^2}$ does not exist.

5. Write an equation for the plane passing through the points (5,0,1), (-2,2,3), and the origin.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is the most totally confusing experience in my life. The professor told us there were these things we definitely had to know for the test, like in my notes I have that she said that the level curvy things are ninety degrees from the direction of greatest increase. And she said we have to know why that's true, but I totally don't have a clue. I looked in the book and it makes no sense at all. She never said anything about it in class, just during the review. So how am I supposed to know why it's true? This is so unfair!"

Explain clearly to Bunny how she could deduce such a conclusion from other things which she should indeed know.

8. It was stated in class that $\left\|\vec{a}\times\vec{b}\right\|$ gives the area of the parallelogram with sides $\vec{a}$ and $\vec{b}$. Explain why this is true. [Feel free to use the fact that $||\vec{a}\times\vec{b}||=||\vec{a}||||\vec{b}||\sin\theta$, even though we didn't actually prove it in class either.]

9. Suppose that you're standing at a point (x_0, y_0) on the graph of $f(x, y)$. In which direction(s) is the slope of the surface equal to half of the greatest slope at that point?

10. Suppose f is a differentiable function of one variable. Show that all tangent planes to the surface $z = x\,f(y/x)$ intersect in a common point (Stewart 5th, p. 978). [Hint$_1$: Warm up with a simple function, like $f(x) = x^2$. Hint$_2$: If you were lucky, where would the common point be?.]

Extra Credit (5 points possible):

Suppose the region inside the triangle with vertices (1,0), (2,0), and (1,1) is rotated around the line $y = x$. Describe the solid formed, and find its volume.