Problem Set 1 Calculus 3 Due 9/30/05

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 5 points. For full credit indicate clearly how you reached your answer. All work must be legible and submitted on clean paper without ragged edges.

1. ***Mathematica* can handle tedious computations for you.**

Working out $\int \frac{1}{x^4+4x^2+3}\,dx$ by hand involves factoring, a *u*-substitution, a partial fractions decomposition, a trigonometric substitution, and then working out a trigonometric integral. Have *Mathematica* work it out for you instead.

2. **The graphics *Mathematica* can generate sometimes answer questions all by themselves.**

Have *Mathematica* generate a graph of $z = x^2 - y^2$. Look at the vertical traces in the regular plot and the level curves in the contour plot and use them to explain why this shape is called a hyperbolic paraboloid.

3. **And sometimes the graphics provide the key to letting you work out a problem.**

[Putz, "The CAS in multivariable calculus"] Use *Mathematica* to investigate the shape of $f(x,y) = \frac{xy^3}{x^2+y^6}$. Figure out what curve of approach to use to show that $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ does not exist.

4. **Sometimes what you can do by hand just doesn't fit together without the right picture.**

With continuous functions of one variable, if there are two critical points you know they're not both maxima. With functions for two or more variables that gets more complicated. Find the critical points of $f(x, y) = -(x^2 - 1)^2 - (x^2y - x - 1)^2$ and classify them as maxima, minima, or saddle points. Now generate a good graph of this surface and use it to explain how the surface can have the maxima it does without the corresponding minima one might expect.