

1. Find the divergence of the vector field $\mathbf{F}(x,y,z) = \langle -z, y^2, x \rangle$.

$$\begin{aligned} \operatorname{div} \vec{\mathbf{F}} &= \Delta \cdot \vec{\mathbf{F}} = \frac{\partial}{\partial x}(-z) + \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(x) \\ &= 0 + 2y + 0 \\ &= 2y \quad \blacksquare \end{aligned}$$

Careful

2. Find the curl of the vector field $\mathbf{F}(x,y,z) = \langle -z, y^2, x \rangle$.

$$\begin{aligned} \operatorname{curl} \vec{\mathbf{F}} &= \nabla \times \vec{\mathbf{F}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -z & y^2 & x \end{vmatrix} \\ &= -\vec{j} - \vec{j} = -2\vec{j} \end{aligned}$$

$$\operatorname{curl} \vec{\mathbf{F}} = \langle 0, -2, 0 \rangle$$

Excellent