Exam 4   Calc 2   12/1/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Give the 5th degree MacLaurin polynomial for sin x.

2. Find the sum $\dfrac{1}{3} - \dfrac{1}{9} + \dfrac{1}{27} - \dfrac{1}{81} + \ldots$

3. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n+1}$ converges or diverges.

4. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n+3}$ converges or diverges.

5. Determine whether the series $\sum_{n=0}^{\infty} \dfrac{1}{3+n!}$ converges or diverges.

6. Suppose that you begin with the closed interval [0,1] and divide it into five equal parts, then discard the second and fourth open intervals, i.e. (1/5, 2/5) and (3/5, 4/5). Next take each of the remaining intervals and divide each into five equal parts, discarding the second and fourth open intervals as before. If this process is repeated indefinitely, what is the total length of the intervals removed?

7. For which values of p will the series $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^p}$ converge, and why?

8. Biff is a calculus student at Enormous State University, and he's still having some trouble. Biff says "Dude, this series stuff is killing me. Especially these questions where you're supposed to say what it adds up to, I mean what's up with that? When it's just diverges or converges, it's fifty-fifty, and I like those odds, ya know? But there was this question on our test where we were supposed to say what one over e to the n adds up to, and I did it with the integral test and got 1 over e, so I said 1 over e, and the grader wrote a bunch of bad stuff about how I couldn't use that, but I dunno why."

Explain clearly to Biff what was wrong with using the results of the integral test as he did.

9. Use a power series with at least 3 non-zero terms to approximate $\displaystyle\int_0^{0.1} \frac{1}{1-x^5}\,dx$.

10. The radius of convergence of the power series $\sum_{n=2}^{\infty} \dfrac{x^n}{\ln n}$ is 1. Are the endpoints included in the interval of convergence?

Extra Credit (5 points possible):

What is the exact value of $2 + \dfrac{4}{3} + \dfrac{2}{3} + \dfrac{4}{15} + \ldots + \dfrac{2^n}{n!} + \ldots$?